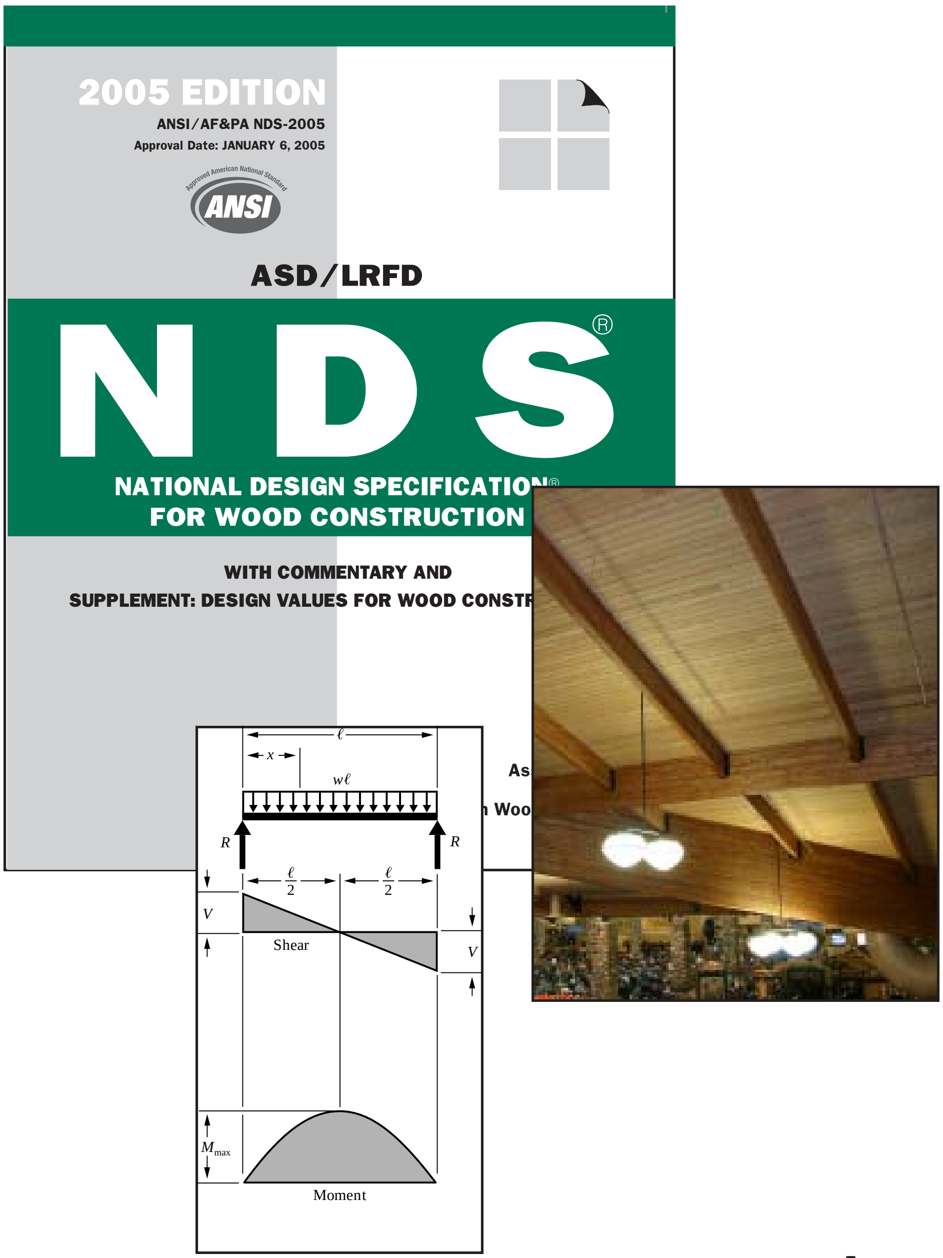
BEAM DESIGN FORMULAS WITH SHEAR AND MOMENT DIAGRAMS



American Wood Council

DESIGN AID No. 6

American Forest & Paper Association

BEAM FORMULAS WITH SHEAR AND MOMENT DIAGRAMS

__The American Wood Council__ (AWC) is part of the wood products group of the American Forest & Paper Association (AF&PA). AF&PA is the national trade association of the forest, paper, and wood products industry, representing member companies engaged in growing, harvesting, and processing wood and wood fiber, manufacturing pulp, paper, and paperboard products from both virgin and recycled fiber, and producing engineered and traditional wood products. For more information see www.afandpa.org.

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Introduction

Figures 1 through 32 provide a series of shear and moment diagrams with accompanying formulas for design of beams under various static loading conditions.

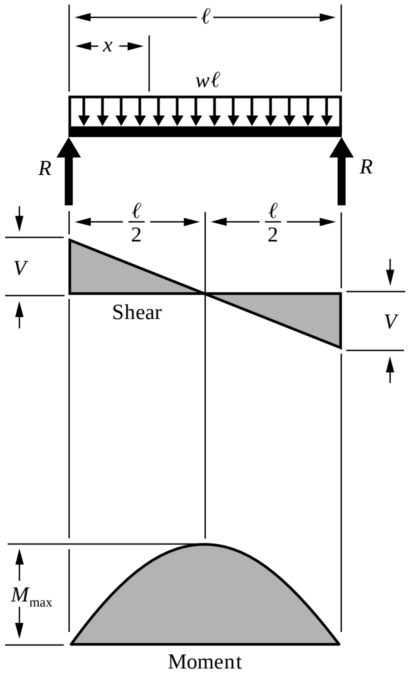
Shear and moment diagrams and formulas are excerpted from the *Western Woods Use Book*, 4th edition, and are provided herein as a courtesy of **Western Wood Products Association**.

Notations Relative to "Shear and Moment Diagrams"

E = modulus of elasticity, psi
I = moment of inertia, in.^{4}
L = span length of the bending member, ft.
ℓ = span length of the bending member, in.
M = maximum bending moment, in.-lbs.
P = total concentrated load, lbs.
R = reaction load at bearing point, lbs.
V = shear force, lbs.
W = total uniform load, lbs.
w = load per unit length, lbs./in.
Δ = deflection or deformation, in.
x = horizontal distance from reaction to point on beam, in.

List of Figures

Figure 1 Simple Beam – Uniformly Distributed Load



$$R = V \quad \ldots\ldots\ldots\ldots\ldots = \frac{w\ell}{2}$$

$$V_x \quad \ldots\ldots\ldots\ldots\ldots = w\left(\frac{\ell}{2} - x\right)$$

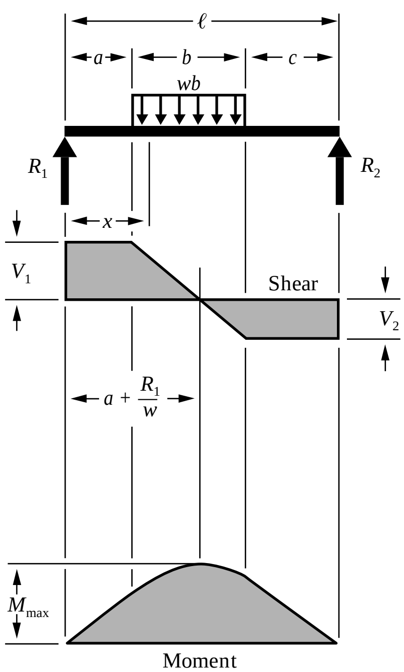
$$M_{max} \text{ (at center)} \quad \ldots\ldots\ldots = \frac{w\ell^2}{8}$$

$$M_x \quad \ldots\ldots\ldots\ldots\ldots = \frac{wx}{2}(\ell - x)$$

$$\Delta_{max} \text{ (at center)} \quad \ldots\ldots\ldots = \frac{5w\ell^4}{384\,EI}$$

$$\Delta_x \quad \ldots\ldots\ldots\ldots\ldots = \frac{wx}{24\,EI}(\ell^3 - 2\ell x^2 + x^3)$$

Figure 2 Simple Beam – Uniform Load Partially Distributed



$$R_1 = V_1 \text{ (max when } a < c) \quad \ldots = \frac{wb}{2\ell}(2c + b)$$

$$R_2 = V_2 \text{ (max when } a > c) \quad \ldots = \frac{wb}{2\ell}(2a + b)$$

$$V_x \left(\text{when } x > a \text{ and } < (a + b)\right) \quad \ldots = R_1 - w(x - a)$$

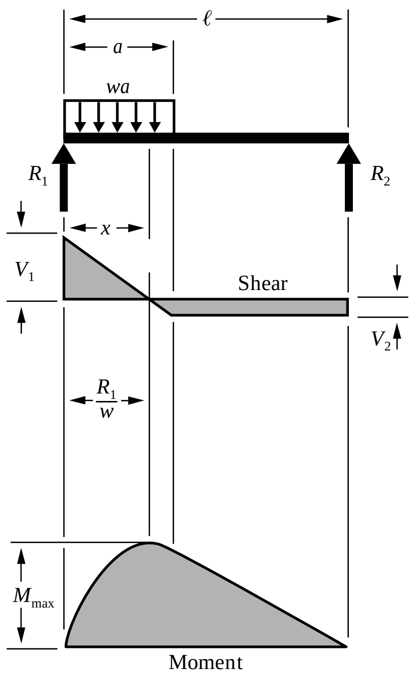
$$M_{max} \left(\text{at } x = a + \frac{R_1}{w}\right) \quad \ldots\ldots = R_1\left(a + \frac{R_1}{2w}\right)$$

$$M_x \text{ (when } x < a) \quad \ldots\ldots\ldots = R_1 x$$

$$M_x \left(\text{when } x > a \text{ and } < (a + b)\right) \quad \ldots = R_1 x - \frac{w}{2}(x - a)^2$$

$$M_x \left(\text{when } x > (a + b)\right) \quad \ldots\ldots = R_2(\ell - x)$$

Figure 3 Simple Beam – Uniform Load Partially Distributed at One End



$$R_1 = V_1 \quad \ldots\ldots\ldots\ldots \quad = \frac{wa}{2\ell}(2\ell - a)$$

$$R_2 = V_2 \quad \ldots\ldots\ldots\ldots \quad = \frac{wa^2}{2\ell}$$

$$V_x \ (\text{when } x < a) \quad \ldots\ldots \quad = R_1 - wx$$

$$M_{max}\left(\text{at } x = \frac{R_1}{w}\right) \quad \ldots\ldots \quad = \frac{R_1^{\,2}}{2w}$$

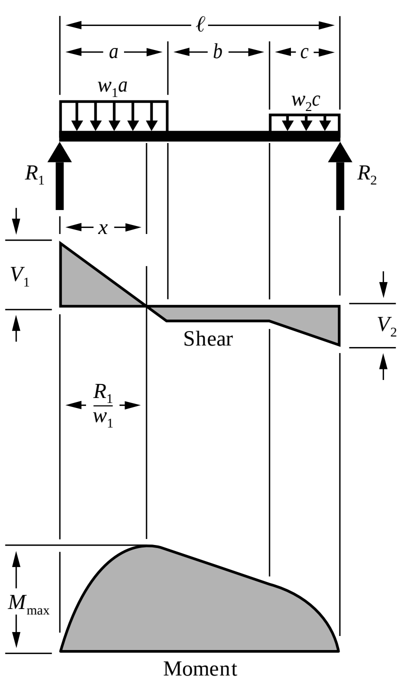
$$M_x \ (\text{when } x < a) \quad \ldots\ldots \quad = R_1 x - \frac{wx^2}{2}$$

$$M_x \ (\text{when } x > a) \quad \ldots\ldots \quad = R_2(\ell - x)$$

$$\Delta_x \ (\text{when } x < a) \quad \ldots\ldots \quad = \frac{wx}{24\,EI\ell}\left(a^2(2\ell - a)^2 - 2ax^2(2\ell - a) + \ell x^3\right)$$

$$\Delta_x \ (\text{when } x > a) \quad \ldots\ldots \quad = \frac{wa^2(\ell - x)}{24\,EI\ell}(4x\ell - 2x^2 - a^2)$$

Figure 4 Simple Beam – Uniform Load Partially Distributed at Each End



$$R_1 = V_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \frac{w_1 a(2\ell - a) + w_2 c^2}{2\ell}$$

$$R_2 = V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \frac{w_2 c(2\ell - c) + w_1 a^2}{2\ell}$$

$$V_x \ (\text{when } x < a) \quad \ldots\ldots\ldots\ldots \quad = R_1 - w_1 x$$

$$V_x \ \left(\text{when } x > a \text{ and } < (a + b)\right) \quad \ldots \quad = R_1 - w_1 a$$

$$V_x \ \left(\text{when } x > (a + b)\right) \quad \ldots\ldots \quad = R_2 - w_2(\ell - x)$$

$$M_{max}\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1 a\right) \ldots \quad = \frac{R_1^{\,2}}{2w_1}$$

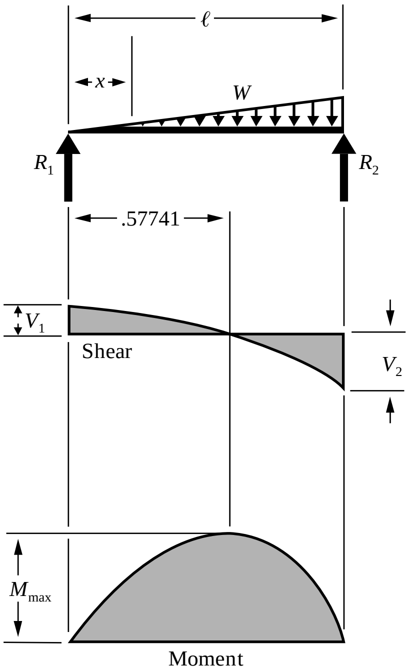
$$M_{max}\left(\text{at } x = \ell - \frac{R_2}{w_2} \text{ when } R_2 < w_2 c\right) \quad = \frac{R_2^{\,2}}{2w_2}$$

$$M_x \ (\text{when } x < a) \quad \ldots\ldots\ldots\ldots \quad = R_1 x - \frac{w_1 x^2}{2}$$

$$M_x \ \left(\text{when } x > a \text{ and } < (a + b)\right) \ldots \quad = R_1 x - \frac{w_1 a}{2}(2x - a)$$

$$M_x \ \left(\text{when } x > (a + b)\right) \quad \ldots\ldots \quad = R_2(\ell - x) - \frac{w_2(\ell - x)^2}{2}$$

Figure 5 Simple Beam – Load Increasing Uniformly to One End



$$R_1 = V_1 \ldots\ldots = \frac{W}{3}$$

$$R_2 = V_2 \ldots\ldots = \frac{2W}{3}$$

$$V_x \ldots\ldots = \frac{W}{3} - \frac{Wx^2}{\ell^2}$$

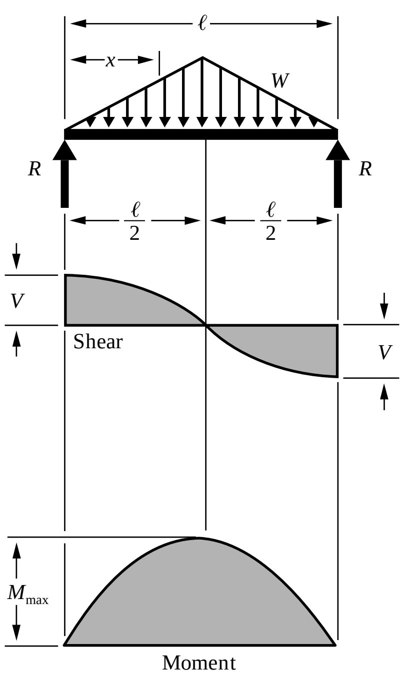
$$M_{max}\left(\text{at } x = \frac{\ell}{\sqrt{3}} = .5774\ell\right) \ldots = \frac{2W\ell}{9\sqrt{3}} = .1283W\ell$$

$$M_x \ldots\ldots = \frac{Wx}{3\ell^2}(\ell^2 - x^2)$$

$$\Delta_{max}\left(\text{at } x = \ell\sqrt{1-\sqrt{\frac{8}{15}}} = .5193\ell\right) . = .01304\frac{W\ell^3}{EI}$$

$$\Delta_x \ldots\ldots = \frac{Wx}{180EI\ell^2}(3x^4 - 10\ell^2x^2 + 7\ell^4)$$

Figure 6 Simple Beam – Load Increasing Uniformly to Center



$$R = V \ldots\ldots = \frac{W}{2}$$

$$V_x\left(\text{when } x < \frac{\ell}{2}\right) \ldots = \frac{W}{2\ell^2}(\ell^2 - 4x^2)$$

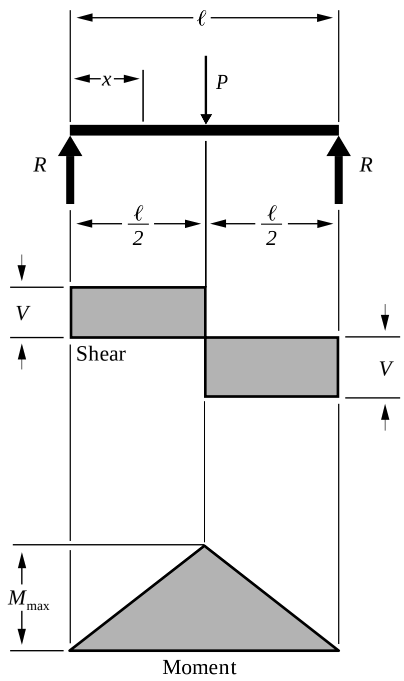
$$M_{max} \text{ (at center)} \ldots = \frac{W\ell}{6}$$

$$M_x\left(\text{when } x < \frac{\ell}{2}\right) \ldots = Wx\left(\frac{1}{2} - \frac{2x^2}{3\ell^2}\right)$$

$$\Delta_{max} \text{ (at center)} \ldots = \frac{W\ell^3}{60EI}$$

$$\Delta_x \ldots\ldots = \frac{Wx}{480EI\ell^2}(5\ell^2 - 4x^2)^2$$

Figure 7 Simple Beam – Concentrated Load at Center



$$R = V \quad \ldots\ldots\ldots\ldots \quad = \frac{P}{2}$$

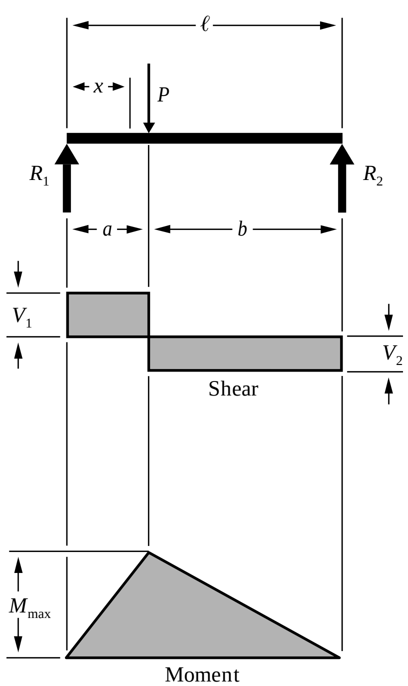
$$M_{max} \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{P\ell}{4}$$

$$M_x \left(\text{when } x < \frac{\ell}{2}\right) \quad \ldots\ldots \quad = \frac{Px}{2}$$

$$\Delta_{max} \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{P\ell^3}{48EI}$$

$$\Delta_x \left(\text{when } x < \frac{\ell}{2}\right) \quad \ldots\ldots \quad = \frac{Px}{48EI}(3\ell^2 - 4x^2)$$

Figure 8 Simple Beam – Concentrated Load at Any Point



$$R_1 = V_1 \text{ (max when } a < b) \quad \ldots\ldots \quad = \frac{Pb}{\ell}$$

$$R_2 = V_2 \text{ (max when } a > b) \quad \ldots\ldots \quad = \frac{Pa}{\ell}$$

$$M_{max} \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{Pab}{\ell}$$

$$M_x \text{ (when } x < b) \quad \ldots\ldots \quad = \frac{Pbx}{\ell}$$

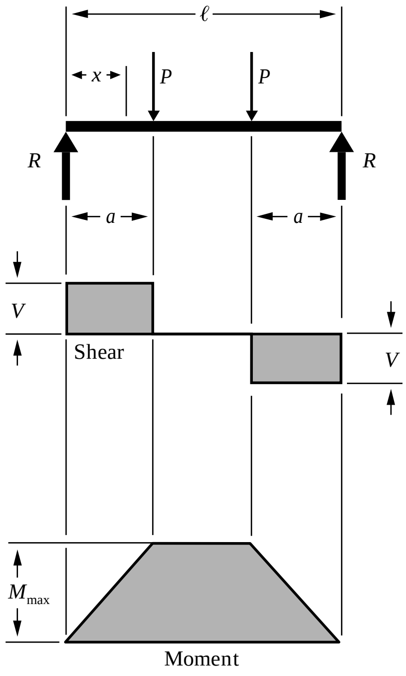
$$\Delta_{max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) \quad . \quad = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$$

$$\Delta_a \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{Pa^2b^2}{3EI\ell}$$

$$\Delta_x \text{ (when } x < a) \quad \ldots\ldots \quad = \frac{Pbx}{6EI\ell}(\ell^2 - b^2 - x^2)$$

$$\Delta_x \text{ (when } x > a) \quad \ldots\ldots \quad = \frac{Pa(\ell - x)}{6EI\ell}(2\ell x - x^2 - a^2)$$

Figure 9 Simple Beam – Two Equal Concentrated Loads Symmetrically Placed



$R = V$ . . . . $= P$

M_{max} (between loads) . . . . $= Pa$

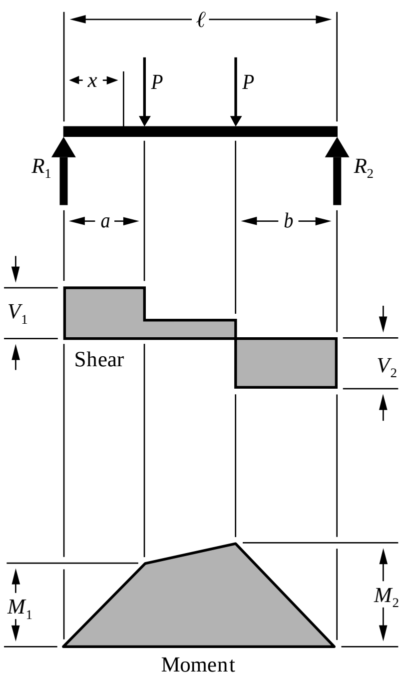
M_x (when $x < a$) . . . . $= Px$

Δ_{max} (at center) . . . . $= \frac{Pa}{24EI}(3\ell^2 - 4a^2)$

Δ_x (when $x < a$) . . . . $= \frac{Px}{6EI}(3\ell a - 3a^2 - x^2)$

$\Delta_x \left(\text{when } x > a \text{ and } < (\ell - a)\right)$ . . . $= \frac{Pa}{6EI}(3\ell x - 3x^2 - a^2)$

Figure 10 Simple Beam – Two Equal Concentrated Loads Unsymmetrically Placed



$R_1 = V_1$ (max when $a < b$) . . . . . $= \frac{P}{\ell}(\ell - a + b)$

$R_2 = V_2$ (max when $a > b$) . . . . . $= \frac{P}{\ell}(\ell - b + a)$

$V_x \left(\text{when } x > a \text{ and } < (\ell - b)\right)$ . . . $= \frac{P}{\ell}(b - a)$

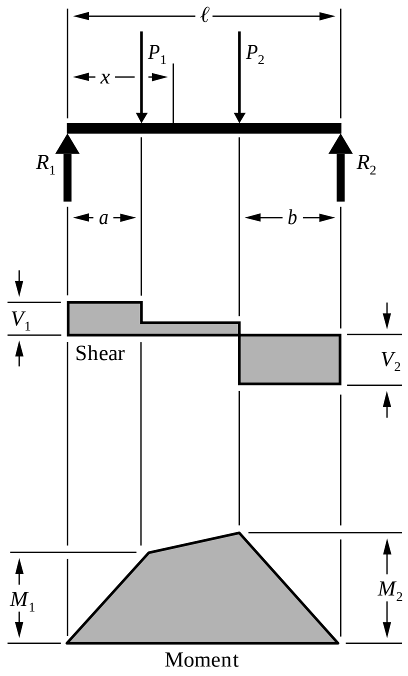
M_1 (max when $a > b$) . . . . $= R_1 a$

M_2 (max when $a < b$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

$M_x \left(\text{when } x > a \text{ and } < (\ell - b)\right)$ . . . $= R_1 x - P(x - a)$

Figure 11 Simple Beam – Two Unequal Concentrated Loads Unsymmetrically Placed



$R_1 = V_1$ . . . . $= \dfrac{P_1(\ell - a) + P_2 b}{\ell}$

$R_2 = V_2$ . . . . $= \dfrac{P_1 a + P_2(\ell - b)}{\ell}$

V_x $\left(\text{when } x > a \text{ and } < (\ell - b)\right)$ . . . $= R_1 - P_1$

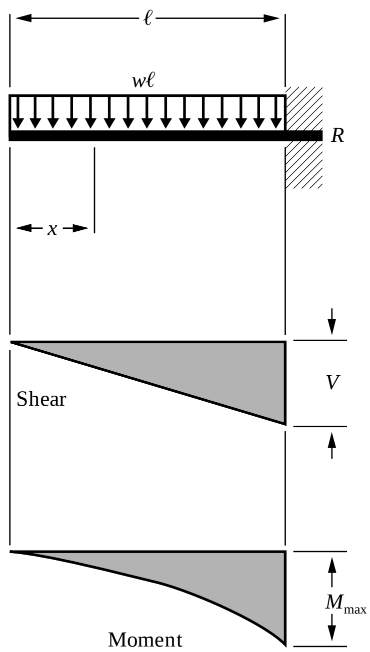
M_1 (max when $R_1 < P_1$) . . . . $= R_1 a$

M_2 (max when $R_2 < P_2$) . . . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x $\left(\text{when } x > a \text{ and } < (\ell - b)\right)$ . . . $= R_1 x - P_1(x - a)$

Figure 12 Cantilever Beam – Uniformly Distributed Load



$R = V$ . . . . $= w\ell$

V_x . . . . $= wx$

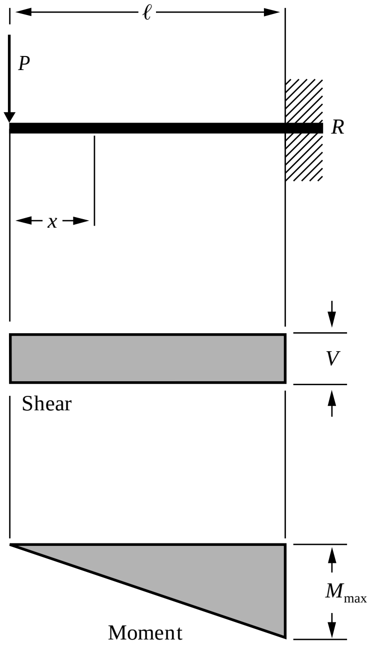
M_{max} (at fixed end) . . . . $= \dfrac{w\ell^2}{2}$

M_x . . . . $= \dfrac{wx^2}{2}$

Δ_{max} (at free end) . . . . $= \dfrac{w\ell^4}{8EI}$

Δ_x . . . . $= \dfrac{w}{24EI}(x^4 - 4\ell^3 x + 3\ell^4)$

Figure 13 Cantilever Beam – Concentrated Load at Free End



$R = V$ . . . . $= P$

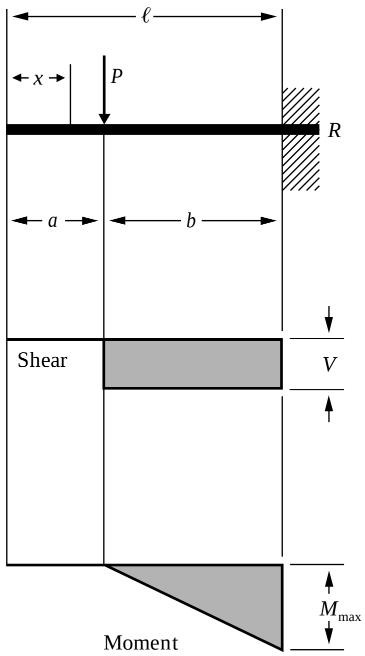
M_{max} (at fixed end) . . . . $= P\ell$

M_x . . . . $= Px$

Δ_{max} (at free end) . . . . $= \dfrac{P\ell^3}{3EI}$

Δ_x . . . . $= \dfrac{P}{6EI}(2\ell^3 - 3\ell^2 x + x^3)$

Figure 14 Cantilever Beam – Concentrated Load at Any Point



$R = V$ . . . . $= P$

M_{max} (at fixed end) . . . . $= Pb$

M_x (when $x > a$) . . . . $= P(x - a)$

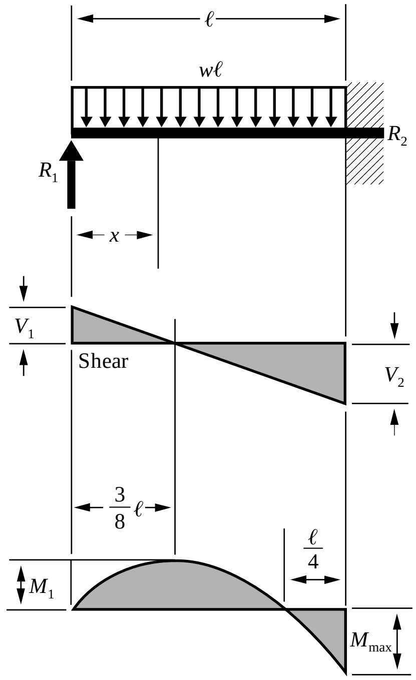
Δ_{max} (at free end) . . . . $= \dfrac{Pb^2}{6EI}(3\ell - b)$

Δ_a (at point of load) . . . . $= \dfrac{Pb^3}{3EI}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pb^2}{6EI}(3\ell - 3x - b)$

Δ_x (when $x > a$) . . . . $= \dfrac{P(\ell - x)^2}{6EI}(3b - \ell + x)$

Figure 15 Beam Fixed at One End, Supported at Other – Uniformly Distributed Load



$$R_1 = V_1 \ldots = \frac{3w\ell}{8}$$

$$R_2 = V_2 \ldots = \frac{5w\ell}{8}$$

$$V_x \ldots = R_1 - wx$$

$$M_{max} \ldots = \frac{w\ell^2}{8}$$

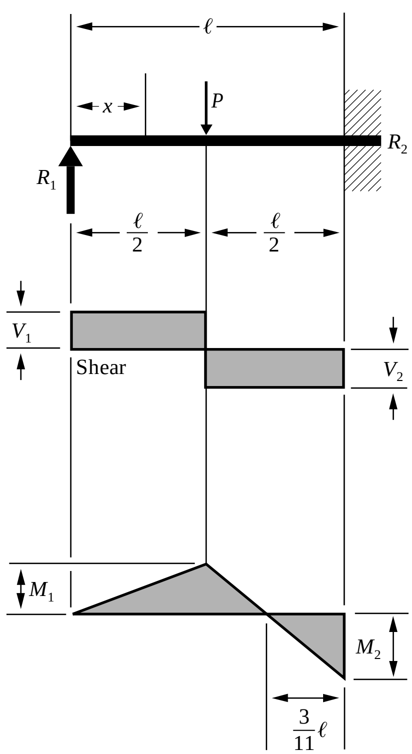
$$M_1\left(\text{at } x = \frac{3}{8}\ell\right) \ldots = \frac{9}{128}w\ell^2$$

$$M_x \ldots = R_1 x - \frac{wx^2}{2}$$

$$\Delta_{max}\left(\text{at } x = \frac{\ell}{16}(1+\sqrt{33}) = .4215\,\ell\right) \ldots = \frac{w\ell^4}{185EI}$$

$$\Delta_x \ldots = \frac{wx}{48EI}(\ell^3 - 3\ell x^2 + 2x^3)$$

Figure 16 Beam Fixed at One End, Supported at Other – Concentrated Load at Center



$$R_1 = V_1 \ldots = \frac{5P}{16}$$

$$R_2 = V_2 \ldots = \frac{11P}{16}$$

$$M_{max} \text{ (at fixed end)} \ldots = \frac{3P\ell}{16}$$

$$M_1 \text{ (at point of load)} \ldots = \frac{5P\ell}{32}$$

$$M_x\left(\text{when } x < \frac{\ell}{2}\right) \ldots = \frac{5Px}{16}$$

$$M_x\left(\text{when } x > \frac{\ell}{2}\right) \ldots = P\left(\frac{\ell}{2} - \frac{11x}{16}\right)$$

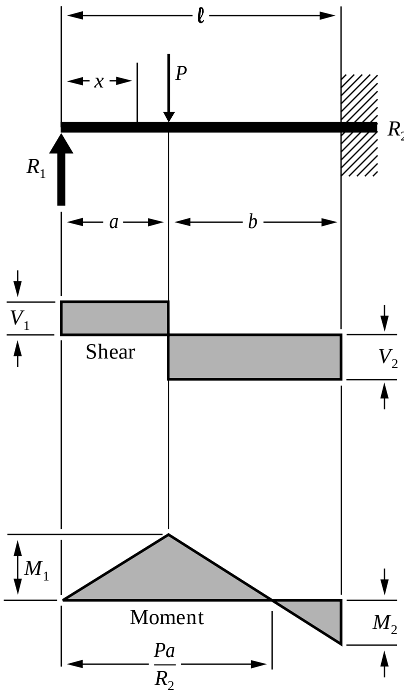
$$\Delta_{max}\left(\text{at } x = \ell\sqrt{\frac{1}{5}} = .4472\ell\right) \ldots = \frac{P\ell^3}{48EI\sqrt{5}} = .009317\frac{P\ell^3}{EI}$$

$$\Delta_x \text{ (at point of load)} \ldots = \frac{7P\ell^3}{768EI}$$

$$\Delta_x\left(\text{when } x < \frac{\ell}{2}\right) \ldots = \frac{Px}{96EI}(3\ell^2 - 5x^2)$$

$$\Delta_x\left(\text{when } x > \frac{\ell}{2}\right) \ldots = \frac{P}{96EI}(x-\ell)^2(11x - 2\ell)$$

Figure 17 Beam Fixed at One End, Supported at Other – Concentrated Load at Any Point



$$R_1 = V_1 \;\ldots\ldots\ldots\ldots\ldots\; = \frac{Pb^2}{2\ell^3}(a + 2\ell)$$

$$R_2 = V_2 \;\ldots\ldots\ldots\ldots\ldots\; = \frac{Pa}{2\ell^3}(3\ell^2 - a^2)$$

$$M_1 \text{ (at point of load) } \ldots\ldots\ldots = R_1 a$$

$$M_2 \text{ (at fixed end) } \ldots\ldots\ldots = \frac{Pab}{2\ell^2}(a + \ell)$$

$$M_x \text{ (when } x < a) \;\ldots\ldots\ldots = R_1 x$$

$$M_x \text{ (when } x > a) \;\ldots\ldots\ldots = R_1 x - P(x - a)$$

$$\Delta_{max}\left(\text{when } a < .414\ell \text{ at } x = \ell\,\frac{\ell^2 + a^2}{3\ell^2 - a^2}\right) = \frac{Pa}{3EI}\,\frac{(\ell^2 - a^2)^3}{(3\ell^2 - a^2)^2}$$

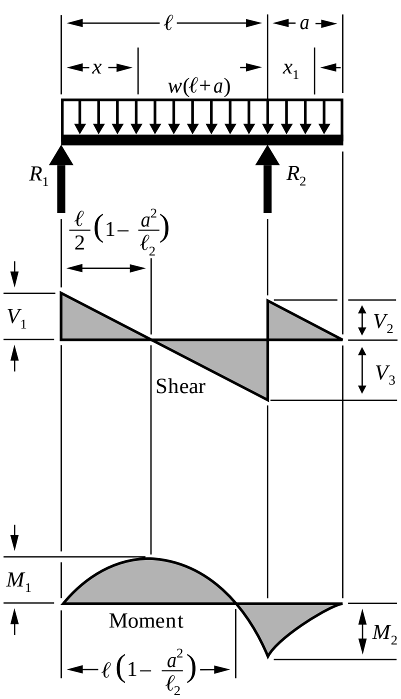
$$\Delta_{max}\left(\text{when } a > .414\ell \text{ at } x = \ell\sqrt{\frac{a}{2\ell + a}}\right) = \frac{Pab^2}{6EI}\sqrt{\frac{a}{2\ell + a}}$$

$$\Delta_a \text{ (at point of load) } \ldots\ldots\ldots = \frac{Pa^2 b^3}{12EI\ell^3}(3\ell + a)$$

$$\Delta_x \text{ (when } x < a) \;\ldots\ldots\ldots = \frac{Pb^2 x}{12EI\ell^3}(3a\ell^2 - 2\ell x^2 - ax^2)$$

$$\Delta_x \text{ (when } x > a) \;\ldots\ldots\ldots = \frac{Pa}{12EI\ell^3}(\ell - x)^2(3\ell^2 x - a^2 x - 2a^2\ell)$$

Figure 18 Beam Overhanging One Support – Uniformly Distributed Load



$$R_1 = V_1 \;\ldots\ldots\ldots\ldots\; = \frac{w}{2\ell}(\ell^2 - a^2)$$

$$R_2 = V_2 + V_3 \;\ldots\ldots\ldots\; = \frac{w}{2\ell}(\ell + a)^2$$

$$V_2 \;\ldots\ldots\ldots\ldots\ldots\; = wa$$

$$V_3 \;\ldots\ldots\ldots\ldots\ldots\; = \frac{w}{2\ell}(\ell^2 + a^2)$$

$$V_x \text{ (between supports) } \ldots = R_1 - wx$$

$$V_{x_1} \text{ (for overhang) } \ldots\ldots = w(a - x_1)$$

$$M_1\left(\text{at } x = \frac{\ell}{2}\left[1 - \frac{a^2}{\ell^2}\right]\right) \ldots = \frac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$$

$$M_2 \text{ (at } R_2) \;\ldots\ldots\ldots = \frac{wa^2}{2}$$

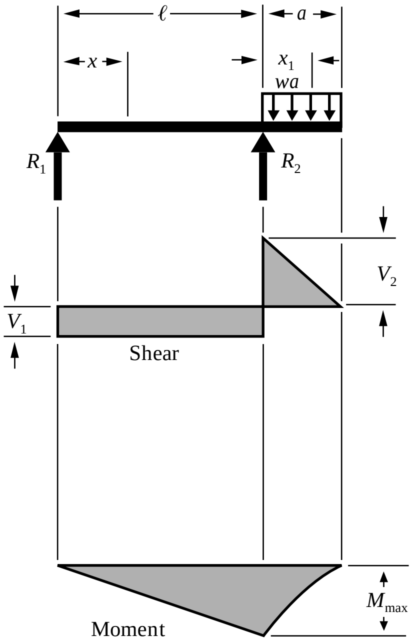
$$M_x \text{ (between supports) } \ldots = \frac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$$

$$M_{x_1} \text{ (for overhang) } \ldots\ldots = \frac{w}{2}(a - x_1)^2$$

$$\Delta_x \text{ (between supports) } \ldots = \frac{wx}{24EI\ell}(\ell^4 - 2\ell^2 x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2 x^2)$$

$$\Delta_{x_1} \text{ (for overhang) } \ldots\ldots = \frac{wx_1}{24EI}(4a^2\ell - \ell^3 + 6a^2 x_1 - 4ax_1^2 + x_1^3)$$

Figure 19 Beam Overhanging One Support – Uniformly Distributed Load on Overhang



$$R_1 = V_1 \quad \ldots\ldots\ldots\ldots \quad = \frac{wa^2}{2\ell}$$

$$R_2 = V_1 + V_2 \quad \ldots\ldots\ldots\ldots \quad = \frac{wa}{2\ell}(2\ell + a)$$

$$V_2 \quad \ldots\ldots\ldots\ldots \quad = wa$$

$$V_{x_1} \text{ (for overhang)} \quad \ldots\ldots\ldots\ldots \quad = w(a - x_1)$$

$$M_{max} \text{ (at } R_2) \quad \ldots\ldots\ldots\ldots \quad = \frac{wa^2}{2}$$

$$M_x \text{ (between supports)} \quad \ldots\ldots\ldots\ldots \quad = \frac{wa^2 x}{2\ell}$$

$$M_{x_1} \text{ (for overhang)} \quad \ldots\ldots\ldots\ldots \quad = \frac{w}{2}(a - x_1)^2$$

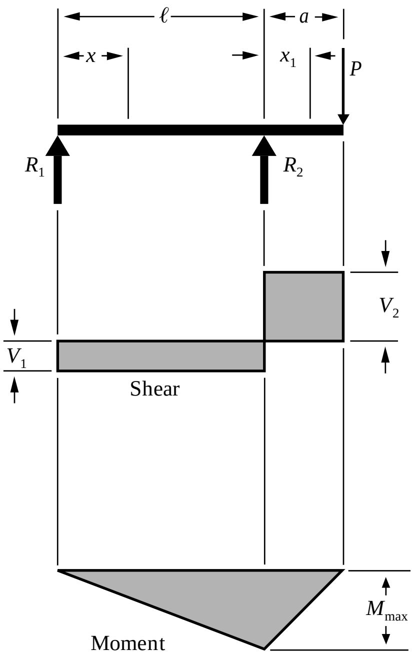
$$\Delta_{max} \left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right) = \frac{wa^2\ell^2}{18\sqrt{3}EI} = .03208\,\frac{wa^2\ell^2}{EI}$$

$$\Delta_{max} \text{ (for overhang at } x_1 = a) \quad \ldots \quad = \frac{wa^3}{24EI}(4\ell + 3a)$$

$$\Delta_x \text{ (between supports)} \quad \ldots\ldots\ldots\ldots \quad = \frac{wa^2 x}{12EI\ell}(\ell^2 - x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} \quad \ldots\ldots\ldots\ldots \quad = \frac{wx_1}{24EI}(4a^2\ell + 6a^2x_1 - 4ax_1^2 + x_1^3)$$

Figure 20 Beam Overhanging One Support – Concentrated Load at End of Overhang



$$R_1 = V_1 \quad \ldots\ldots\ldots\ldots \quad = \frac{Pa}{\ell}$$

$$R_2 = V_1 + V_2 \quad \ldots\ldots\ldots\ldots \quad = \frac{P}{\ell}(\ell + a)$$

$$V_2 \quad \ldots\ldots\ldots\ldots \quad = P$$

$$M_{max} \text{ (at } R_2) \quad \ldots\ldots\ldots\ldots \quad = Pa$$

$$M_x \text{ (between supports)} \quad \ldots\ldots\ldots\ldots \quad = \frac{Pax}{\ell}$$

$$M_{x_1} \text{ (for overhang)} \quad \ldots\ldots\ldots\ldots \quad = P(a - x_1)$$

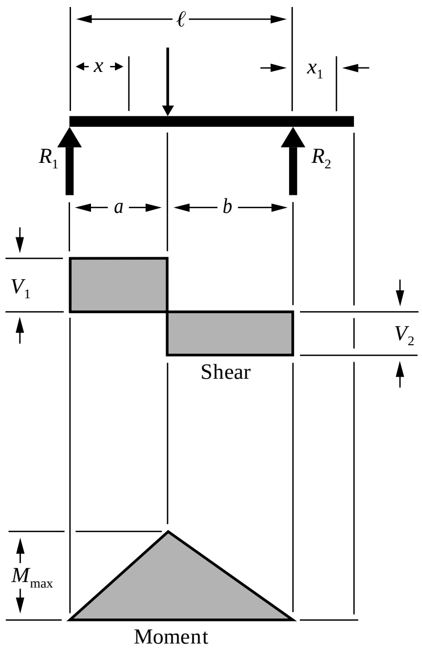
$$\Delta_{max} \left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right) = \frac{Pa\ell^2}{9\sqrt{3}EI} = .06415\,\frac{Pa\ell^2}{EI}$$

$$\Delta_{max} \text{ (for overhang at } x_1 = a) \quad \ldots \quad = \frac{Pa^2}{3EI}(\ell + a)$$

$$\Delta_x \text{ (between supports)} \quad \ldots\ldots\ldots\ldots \quad = \frac{Pax}{6EI\ell}(\ell^2 - x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} \quad \ldots\ldots\ldots\ldots \quad = \frac{Px_1}{6EI}(2a\ell + 3ax_1 - x_1^2)$$

Figure 21 Beam Overhanging One Support – Concentrated Load at Any Point Between Supports



$$R_1 = V_1 \text{ (max when } a < b) \; \ldots\ldots \; = \frac{Pb}{\ell}$$

$$R_2 = V_2 \text{ (max when } a > b) \; \ldots\ldots \; = \frac{Pa}{\ell}$$

$$M_{max} \text{ (at point of load)} \; \ldots\ldots \; = \frac{Pab}{\ell}$$

$$M_x \text{ (when } x < a) \; \ldots\ldots \; = \frac{Pbx}{\ell}$$

$$\Delta_{max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) \; . \; = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$$

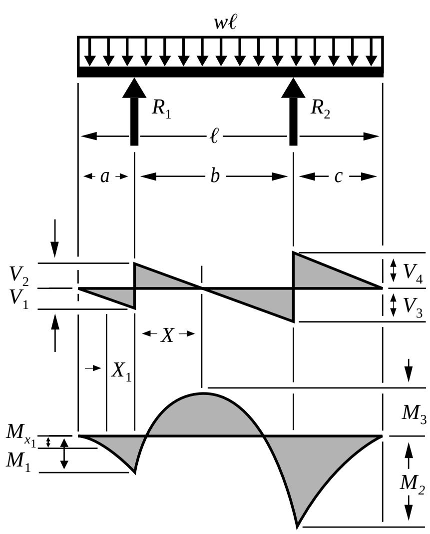
$$\Delta_a \text{ (at point of load)} \; \ldots\ldots \; = \frac{Pa^2b^2}{3EI\ell}$$

$$\Delta_x \text{ (when } x < a) \; \ldots\ldots \; = \frac{Pbx}{6EI\ell}(\ell^2 - b^2 - x^2)$$

$$\Delta_x \text{ (when } x > a) \; \ldots\ldots \; = \frac{Pa(\ell - x)}{6EI\ell}(2\ell x - x^2 - a^2)$$

$$\Delta_{x_1} \; \ldots\ldots \; = \frac{Pabx_1}{6EI\ell}(\ell + a)$$

Figure 22 Beam Overhanging Both Supports – Unequal Overhangs – Uniformly Distributed Load



$$R_1 \; \ldots\ldots \; = \frac{w\ell\,(\ell - 2c)}{2b}$$

$$R_2 \; \ldots\ldots \; = \frac{w\ell\,(\ell - 2a)}{2b}$$

$$V_1 \; \ldots\ldots \; = wa$$

$$V_2 \; \ldots\ldots \; = R_1 - V_1$$

$$V_3 \; \ldots\ldots \; = R_2 - V_4$$

$$V_4 \; \ldots\ldots \; = wc$$

$$V_{x_1} \; \ldots\ldots \; = V_1 - wx_1$$

$$V_x \text{ (when } x < \ell) \; \ldots\ldots \; = R_1 - w(a + x_1)$$

$$V_m \text{ (when } a < c) \; \ldots\ldots \; = R_2 - wc$$

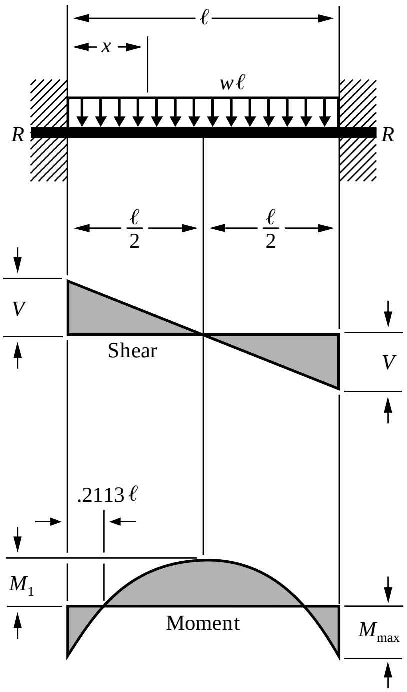
$$M_1 \; \ldots\ldots \; = -\frac{wa^2}{2}$$

$$M_2 \; \ldots\ldots \; = -\frac{wc^2}{2}$$

$$M_3 \; \ldots\ldots \; = R_1\left(\frac{R_1}{2w} - a\right)$$

$$M_x\left(\text{max when } x = \frac{R_1}{w} - a\right) \; \ldots\ldots \; = R_1 x - \frac{w(a+x)^2}{2}$$

Figure 23 Beam Fixed at Both Ends – Uniformly Distributed Load



$R = V$ . . . . $= \frac{w\ell}{2}$

V_x . . . . $= w\left(\frac{\ell}{2} - x\right)$

M_{max} (at ends) . . . . $= \frac{w\ell^2}{12}$

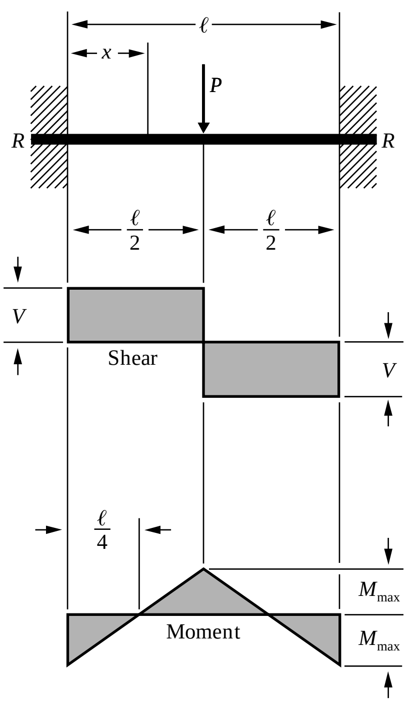
M_1 (at center) . . . . $= \frac{w\ell^2}{24}$

M_x . . . . $= \frac{w}{12}(6\ell x - \ell^2 - 6x^2)$

Δ_{max} (at center) . . . . $= \frac{w\ell^4}{384EI}$

Δ_x . . . . $= \frac{wx^2}{24EI}(\ell - x)^2$

Figure 24 Beam Fixed at Both Ends – Concentrated Load at Center



$R = V$ . . . . $= \frac{P}{2}$

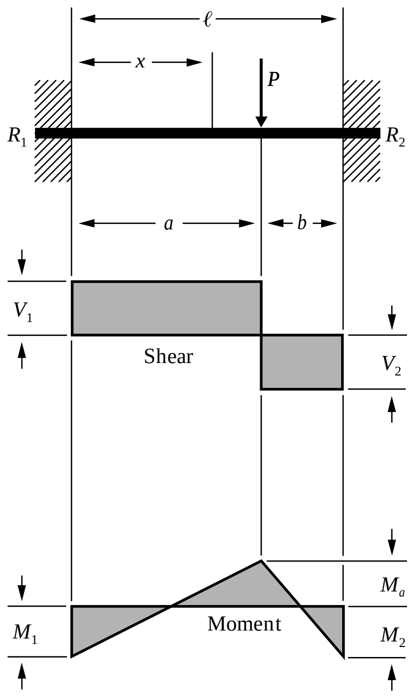
M_{max} (at center and ends) . . . . $= \frac{P\ell}{8}$

$M_x \left(\text{when } x < \frac{\ell}{2}\right)$ . . . . $= \frac{P}{8}(4x - \ell)$

Δ_{max} (at center) . . . . $= \frac{P\ell^3}{192EI}$

$\Delta_x \left(\text{when } x < \frac{\ell}{2}\right)$ . . . . $= \frac{Px^2}{48EI}(3\ell - 4x)$

Figure 25 Beam Fixed at Both Ends – Concentrated Load at Any Point



$R_1 = V_1$ (max when $a < b$) . . . . . . $= \dfrac{Pb^2}{\ell^3}(3a + b)$

$R_2 = V_2$ (max when $a > b$) . . . . . . $= \dfrac{Pa^2}{\ell^3}(a + 3b)$

M_1 (max when $a < b$) . . . . $= \dfrac{Pab^2}{\ell^2}$

M_2 (max when $a > b$) . . . . $= \dfrac{Pa^2b}{\ell^2}$

M_a (at point of load) . . . . $= \dfrac{2Pa^2b^2}{\ell^3}$

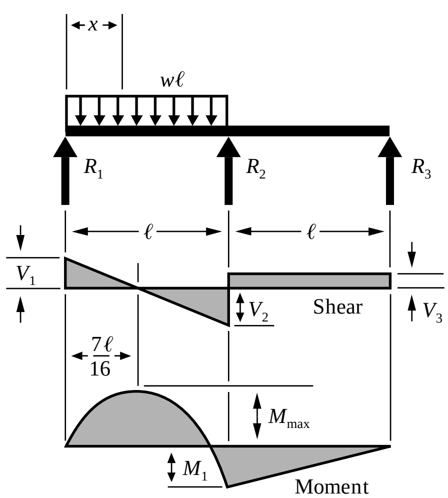
M_x (when $x < a$) . . . . $= R_1x - \dfrac{Pab^2}{\ell^2}$

$\Delta_{max}\left(\text{when } a > b \text{ at } x = \dfrac{2a\ell}{3a + b}\right)$ . . . $= \dfrac{2Pa^3b^2}{3EI(3a + b)^2}$

Δ_a (at point of load) . . . . $= \dfrac{Pa^3b^3}{3EI\ell^3}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pb^2x^2}{6EI\ell^3}(3a\ell - 3ax - bx)$

Figure 26 Continuous Beam – Two Equal Spans – Uniform Load on One Span



$R_1 = V_1$ . . . . $= \dfrac{7}{16}w\ell$

$R_2 = V_2 + V_3$ . . . . $= \dfrac{5}{8}w\ell$

$R_3 = V_3$ . . . . $= -\dfrac{1}{16}w\ell$

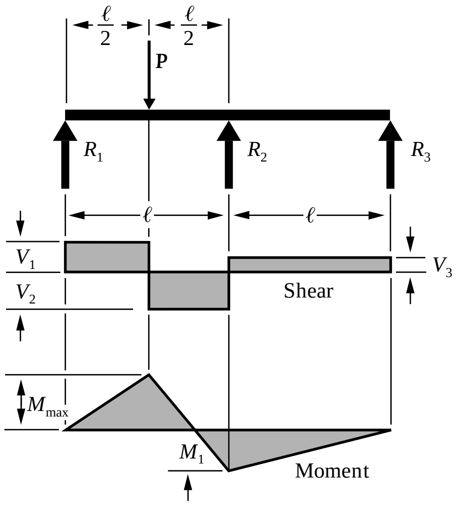
V_2 . . . . $= \dfrac{9}{16}w\ell$

$M_{max}\left(\text{at } x = \dfrac{7}{16}\ell\right)$ . . . . $= \dfrac{49}{512}w\ell^2$

M_1 (at support R_2) . . . . $= \dfrac{1}{16}w\ell^2$

M_x (when $x < \ell$) . . . . $= \dfrac{wx}{16}(7\ell - 8x)$

Figure 27 Continuous Beam – Two Equal Spans – Concentrated Load at Center of One Span



$$R_1 = V_1 \quad \dots\dots\dots\dots\dots \quad = \frac{13}{32}P$$

$$R_2 = V_2 + V_3 \quad \dots\dots\dots\dots \quad = \frac{11}{16}P$$

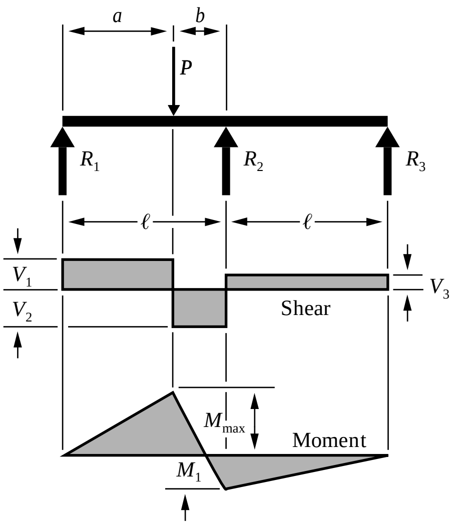
$$R_3 = V_3 \quad \dots\dots\dots\dots\dots \quad = -\frac{3}{32}P$$

$$V_2 \quad \dots\dots\dots\dots\dots\dots \quad = \frac{19}{32}P$$

$$M_{max} \text{ (at point of load)} \quad \dots\dots\dots \quad = \frac{13}{64}P\ell$$

$$M_1 \text{ (at support } R_2\text{)} \quad \dots\dots\dots\dots \quad = \frac{3}{32}P\ell$$

Figure 28 Continuous Beam – Two Equal Spans – Concentrated Load at Any Point



$$R_1 = V_1 \quad \dots\dots\dots\dots\dots \quad = \frac{Pb}{4\ell^3}\left(4\ell^2 - a(\ell + a)\right)$$

$$R_2 = V_2 + V_3 \quad \dots\dots\dots\dots \quad = \frac{Pa}{2\ell^3}\left(2\ell^2 + b(\ell + a)\right)$$

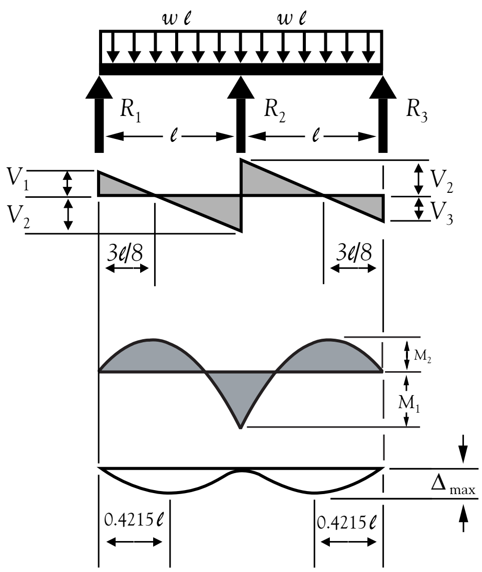
$$R_3 = V_3 \quad \dots\dots\dots\dots\dots \quad = -\frac{Pab}{4\ell^3}(\ell + a)$$

$$V_2 \quad \dots\dots\dots\dots\dots\dots \quad = \frac{Pa}{4\ell^3}\left(4\ell^2 + b(\ell + a)\right)$$

$$M_{max} \text{ (at point of load)} \quad \dots\dots\dots \quad = \frac{Pab}{4\ell^3}\left(4\ell^2 - a(\ell + a)\right)$$

$$M_1 \text{ (at support } R_2\text{)} \quad \dots\dots\dots\dots \quad = \frac{Pab}{4\ell^2}(\ell + a)$$

Figure 29 Continuous Beam – Two Equal Spans – Uniformly Distributed Load



$R_1 = V_1 = R_3 = V_3$ . . . . $= \dfrac{3w\ell}{8}$

R_2 . . . . $= \dfrac{10w\ell}{8}$

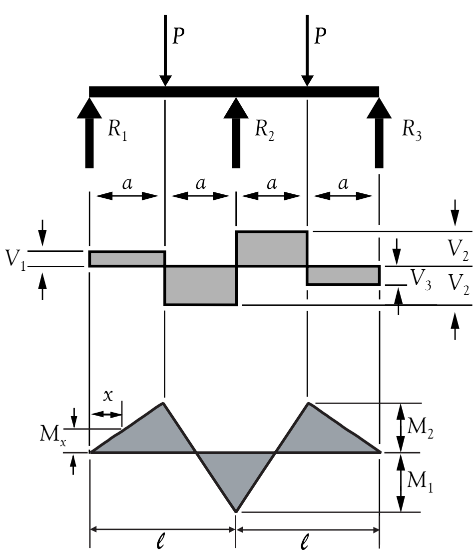
$V_2 = V_{max}$ . . . . $= \dfrac{5w\ell}{8}$

M_1 . . . . $= \dfrac{w\ell^2}{8}$

$M_2 \left(\text{at } \dfrac{3\ell}{8}\right)$ . . . . $= \dfrac{9w\ell^2}{128}$

Δ_{max} (at $0.4215\,\ell$, approx. from R_1 and R_3) . . $= \dfrac{w\ell^4}{185EI}$

Figure 30 Continuous Beam – Two Equal Spans – Two Equal Concentrated Loads Symmetrically Placed



$R_1 = V_1 = R_3 = V_3$ . . . . $= \dfrac{5P}{16}$

$R_2 = 2V_2$ . . . . $= \dfrac{11P}{8}$

$V_2 = P - R_1$ . . . . $= \dfrac{11P}{16}$

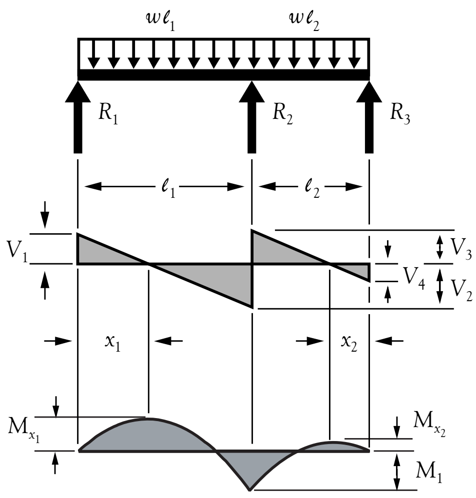
V_{max} . . . . $= V_2$

M_1 . . . . $= -\dfrac{3P\ell}{16}$

M_2 . . . . $= \dfrac{5P\ell}{32}$

M_x (when $x < a$) . . . . $= R_1 x$

Figure 31 Continuous Beam – Two Unequal Spans – Uniformly Distributed Load



R_1 . . . . $= \dfrac{M_1}{\ell_1} + \dfrac{w\ell_1}{2}$

R_2 . . . . $= w\ell_1 + w\ell_2 - R_1 - R_3$

$R_3 = V_4$ . . . . $= \dfrac{M_1}{\ell_2} + \dfrac{w\ell_2}{2}$

V_1 . . . . $= R_1$

V_2 . . . . $= w\ell_1 - R_1$

V_3 . . . . $= w\ell_2 - R_3$

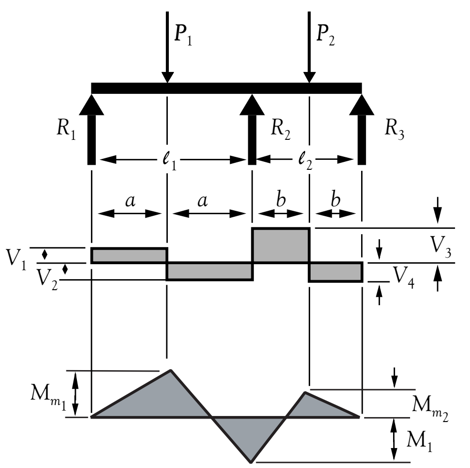
V_4 . . . . $= R_3$

M_1 . . . . $= -\dfrac{w\ell_2^{\,3} + w\ell_1^{\,3}}{8(\ell_1 + \ell_2)}$

$M_{x_1}\left(\text{when } x_1 = \dfrac{R_1}{w}\right)$ . . . . $= R_1 x_1 - \dfrac{w x_1^2}{2}$

$M_{x_2}\left(\text{when } x_2 = \dfrac{R_3}{w}\right)$ . . . . $= R_3 x_2 - \dfrac{w x_2^2}{2}$

Figure 32 Continuous Beam – Two Unequal Spans – Concentrated Load on Each Span Symmetrically Placed



R_1 . . . . $= \dfrac{M_1}{\ell_1} + \dfrac{P_1}{2}$

R_2 . . . . $= P_1 + P_2 - R_1 - R_3$

R_3 . . . . $= \dfrac{M_1}{\ell_2} + \dfrac{P_2}{2}$

V_1 . . . . $= R_1$

V_2 . . . . $= P_1 - R_1$

V_3 . . . . $= P_2 - R_3$

V_4 . . . . $= R_3$

M_1 . . . . $= -\dfrac{3}{16}\left(\dfrac{P_1\ell_1^{\,2} + P_2\ell_2^{\,2}}{\ell_1 + \ell_2}\right)$

M_{m_1} . . . . $= R_1 a$

M_{m_2} . . . . $= R_3 b$

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