Measurement of Permeability

Permeability can be measured in the laboratory and in the field. Each method has its strengths and its weaknesses.

Laboratory tests of permeability do not require drilling pump and observation wells, potentially hundreds of feet deep, but the tests are performed on a small sample. The small sample size can lead to skewed values of permeability because the effect of sand and clay seams is often missed. Permeability tests on fine grained soils can take days or weeks to complete. The long time frame can lead to leaks in the testing apparatus (permeameter or triaxial test cell) which will also skew the results.

Field tests of permeability will generally account for the effect of sand and clay seams, but require drilling pump and monitoring wells and lowering the ground water table over a fairly large area. Disposal of the discharge water could be an issue if contamination is present.

Laboratory Measurement of Permeability

Constant Head Test:

The constant head permeability test is applicable to coarse grain soils. The time to complete the test can be a few seconds to a few minutes, depending upon the amount of fines in the sample. An idealized set-up of a constant head test is shown below.

A constant pressure (head) of water is maintained across the sample by use of an overflow outlet. The cross-sectional area (A) and length (L) of the sample are assumed to be constant throughout the test. The difference between the entering and exiting water levels (h) represent the energy lost as the water flowed through the sample. The volume of water that flowed through the sample (V) in the specified time (t) is collected and measured.

It is important that a consistent set of units is maintained
Idealized constant head permeability set-up.

\[ k = \frac{(V)(L)}{(A)(h)(t)} \]

\( V \) = volume of water passing through specimen (volume or length cubed)
  
  1 ml (milliliter) = 1 cm\(^3\)

\( L \) = length of specimen (length)

\( A \) = area of specimen (length squared)

\( h \) = head drop across specimen (length)

\( t \) = time of reading (time)
Falling Head Test:

The falling head permeability test is intended for fine grain soils. Completion of the test may take hours or days. An idealized set-up of the test is shown below.

![Idealized falling head permeability set-up.](image)

In this case, the water flows upward through the sample and an overflow outlet maintains a constant water level on the top of the specimen. The starting (h1) and ending (h2) heads in the pizometer are measured from the
water level on top of the specimen. The cross-sectional area of the pizometer \( (a) \) is usually much less than the cross-sectional area of the specimen \( (A) \) to allow for greater precision in measuring the head drop over the test period.

As with the constant head test, it is important to maintain a constant set of units

\[
k = 2.303 \left( \frac{a \ L}{A \ t} \right) \log_{10} \left( \frac{h_1}{h_2} \right)
\]

or

\[
k = \left( \frac{a \ L}{A \ t} \right) \ln \left( \frac{h_1}{h_2} \right)
\]

- \( k \) = permeability (length per unit time)
- \( a \) = burette area (length squared)
- \( L \) = length of specimen (length)
- \( A \) = area of specimen (length squared)
- \( h_1 \) = initial head drop across specimen (length)
- \( h_2 \) = head drop across specimen at time of reading (length)
- \( t \) = time of reading (time)

FYI,

\[
\log_e = \text{natural log (ln)}
\]

The log of a number to a given base is the power to which the base must be raised in order to produce the number. For example:

\[
\log_2 (8) = 3 \text{ because } (2)^3 = 8
\]

If no base is given (i.e. log), then assume that the base is 10 (i.e. \( \log_{10} \))

\[
\log (100) = 2 \text{ because } (10)^2 = 100
\]
Exercise 12.

A schematic of a constant head permeability test is shown below. The measured pressures at points A and B are 100 psf and 60 psf, respectively. A volume change burette attached to the apparatus measures a flow volume of $2.2 \text{ in}^3$ over a time interval of 15 minutes.

a. Compute the permeability of the soil specimen in in/sec.
b. Compute the discharge velocity of the water through the specimen.
c. In the specimen has a void ratio $e = 0.5$, compute the seepage velocity of the water through the specimen.

**Given**

$v = 2.2 \text{ in}^3$
$t = 15 \text{ min}$
$\frac{t}{15 \text{ min}} = \frac{1600 \text{ sec}}{900 \text{ sec}}$

**Equations**

$k = \frac{v L}{A h t}$

$A = \frac{\pi d^2}{4}$
$h = \frac{P}{\gamma}$

**Solution**

$A = 0.785 (1.25 \text{ in})^2 = 1.23 \text{ in}^2$

$h_A = \frac{100 \text{ psf}}{62.4 \frac{1\text{b}}{\text{ft}^3}} = 1.60 \text{ ft}$
$h_B = \frac{60 \text{ psf}}{62.4 \frac{1\text{b}}{\text{ft}^3}} = 0.96 \text{ ft}$

$h_A = 19.2 \text{ in}$
$h_B = 11.54 \text{ in}$
(a) Permeability, $K = \frac{(2.2 \text{ in}^3)(3 \text{ in})}{(1.23 \text{ in}^2)(19.2 \text{ in} - 11.54 \text{ in})(900 \text{ sec})}$

$$K = 0.000778 \text{ in/ sec} = 7.78 \times 10^{-4} \text{ in/ sec}$$

(b) Discharge Velocity, $V = K i = K \frac{h}{L}$

$$V = (7.78 \times 10^{-4} \text{ in/ sec}) \left(\frac{19.2 \text{ in} - 11.54 \text{ in}}{3 \text{ in}}\right) = 0.00199$$

$$V = 1.99 \times 10^{-3} \text{ in/ sec}$$

(c) Seepage Velocity, $V_s = V \left(\frac{1+e}{e}\right)$

Void ratio, $e = 0.5$

$$V_s = (1.99 \times 10^{-3} \text{ in/ sec}) \left(\frac{1+0.5}{0.5}\right) = 0.00597$$

$$V_s = 5.97 \times 10^{-3} \text{ in/ sec}$$

References:

CERM 13th edition Sections 21-2; 21-3; 21-4; 35-23

"Geotechnical Notes" (from this site)
Field Measurement of Permeability (k)

Measuring the coefficient of permeability in the field requires the drilling of a pump well and 2 observation wells. The wells must be drilled into the aquifer (i.e. below the surface of the ground water table). Once the wells are drilled a pump is inserted into the pump well and water is withdrawn from the well. As the water is withdrawn the surface of the ground water table moves downward in the pump well and in the observation wells.

The field method formulas are based on geometry and the fact that the tests are conducted under equilibrium conditions. This means that an equal volume of water flows back into the aquifer to replace the water that is pumped out. The levels of drawdown are not measured until the water levels in all 3 wells remain at a constant height for a constant amount of discharge (q).

Confined versus Unconfined Aquifers

An aquifer is a water bearing layer of permeable soil and/or rock. A confined aquifer is sandwiched between layers of impermeable material such as unfractured rock or clay. An unconfined aquifer is bounded by only one layer of impermeable material.

Pump Test in Unconfined Aquifer:

\[
k = \frac{q \ln \left( \frac{r_1}{r_2} \right)}{\pi \left( h_1^2 - h_2^2 \right)}
\]

Where
- \( k \) = coefficient of permeability
- \( q \) = discharge from well (volume or length cubed)
- \( r_1 \) = radial distance to Observation Well 1 (length)
- \( r_2 \) = radial distance to Observation Well 2 (length)
- \( h_1 \) = total head in Observation Well 1 (length)
- \( h_2 \) = total head in Observation Well 2 (length)
- \( r_1 > r_2 \)
Schematic of an unconfined aquifer.

Note that:
1. \( h_1 \) and \( h_2 \) are measured from the top of the impermeable layer.
   a. Problem statements and diagrams often report distances from the top of the ground layer
2. \( r_1 \) and \( r_2 \) are measured from the centerline of the pump well to the centerline of the observation wells.
3. if \( q \) is given in volumetric units (i.e. gallons per minute) it must be converted into length cubed per time units.
   a. 1 cfs \( \approx \) 449 gallons per minute
   b. 1 gallon per minute \( \approx \) 0.002227 cfs

Pump Test in Confined Aquifer:

\[
q \ln \left( \frac{r_1}{r_2} \right) = \frac{k}{2\pi D \left( h_1 - h_2 \right)}
\]
Where

- \( k \) = coefficient of permeability
- \( q \) = discharge from well (volume or length cubed)
- \( r_1 \) = radial distance to Observation Well 1 (length)
- \( r_2 \) = radial distance to Observation Well 2 (length)
- \( h_1 \) = total head in Observation Well 1 (length)
- \( h_2 \) = total head in Observation Well 2 (length)
- \( D \) = thickness of aquifer (length)
- \( r_1 > r_2 \)

Schematic of a confined aquifer.

Note that:

1. the quantity \((h_1 - h_2)\) is the difference in the heights of the water level in the observation wells.
2. \( r_1 \) and \( r_2 \) are measured from the centerline of the pump well to the centerline of the observation wells.
3. if \( q \) is given in volumetric units (i.e. gallons per minute) it must be converted into length cubed per time units.
   a. 1 cfs \( \approx \) 449 gallons per minute
   b. 1 gallon per minute \( \approx \) 0.002227 cfs
Estimates of permeability based on soil classification

Order of magnitude estimates of permeability can be made for in-situ soils based on soil classification. The following table contains information culled from several sources:

<table>
<thead>
<tr>
<th>General Soil Type</th>
<th>Permeability, k, (cm/sec)</th>
<th>Degree of Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean gravel</td>
<td>Greater than 1</td>
<td>Very High</td>
</tr>
<tr>
<td>Gravel with some sand or fines</td>
<td>0.1 to 1</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>(1 to 1x10^-1)</td>
<td></td>
</tr>
<tr>
<td>Sandy gravel</td>
<td>0.1 to 0.001</td>
<td>Medium</td>
</tr>
<tr>
<td>Clean sand (few fines)</td>
<td>(1x10^-1 to 1x10^-3)</td>
<td></td>
</tr>
<tr>
<td>Fine sand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>0.001 to 0.00001</td>
<td>Low</td>
</tr>
<tr>
<td>Dirty sand</td>
<td>(1x10^-3 to 1x10^-5)</td>
<td></td>
</tr>
<tr>
<td>Silty sand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silt</td>
<td>0.00001 to 0.0000001</td>
<td>Very Low</td>
</tr>
<tr>
<td>Silty clay</td>
<td>(1x10^-5 to 1x10^-7)</td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td>Less than 0.0000001</td>
<td>Practically Impermeable</td>
</tr>
<tr>
<td></td>
<td>(&lt;1x10^-7)</td>
<td></td>
</tr>
</tbody>
</table>

0.1 cm/sec ≈ 0.00328084 ft/sec ≈ 19.6 miles/year

Sands and gravels with few fines (silts and clays) will have higher permeability’s than sands and gravels that contain a high percentage of fines. Sand with larger particle sizes will have a higher permeability than fine sand.

The figure on the following page also shows ranges of permeability based on soil type. Information relating to type of testing applicable to obtaining data and potential sources of error are also addressed.
<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Direct testing of soil in its original position (e.g., well points)</th>
<th>Indirect determination of coefficient of permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean gravel</td>
<td>Reliable; little experience required. Range of unstable permeability; much experience necessary for correct interpretation</td>
<td>From the grain size distribution (e.g., Hazen’s formula). Only applicable to clean, cohesionless sands and gravels</td>
</tr>
<tr>
<td>Clean sand, clean sand and gravel mixtures</td>
<td>Reliable; little experience required. Constant Head Permeameter; little experience required.</td>
<td>Horizontal Capillary Test: Very little experience necessary; especially useful for rapid testing of a large number of samples in the field without laboratory facilities.</td>
</tr>
<tr>
<td>Very fine sands, organic and inorganic silts, mixtures of sand, silt, and clay, glacial till, stratified clay deposits, etc.</td>
<td>Fairly reliable; considerable experience necessary (do in triaxial cell)</td>
<td>Computations: from consolidation tests; expensive laboratory equipment and considerable experience required.</td>
</tr>
<tr>
<td>“Impervious” soils e.g., homogeneous clays below zone of weathering</td>
<td>Considerable experience also required in this range.</td>
<td></td>
</tr>
<tr>
<td>Impervious sections of earth dams dikes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pervious sections of dams and dikes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good drainage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Practically impervious</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercise 13.

A pump test is performed in an unconfined aquifer as shown in the sketch below. The well is being pumped at a rate \( q = 1.2 \text{ ft}^3/\text{sec} \). Estimate the permeability of the aquifer.

**Equation:** 
\[
-k = 8 \frac{\ln \left( \frac{r_1}{r_2} \right)}{\pi (h_1^2 - h_2^2)}
\]

**Diagram:**

- Observation Well
- Pump Well
- Observation Well
- 30 ft = \( r_2 \)
- 60 ft = \( r_1 \)
- 22.3 ft
- 18.4 ft
- 100 ft

**Solution:**

\[
k = \frac{(1.2 \text{ ft}^3/\text{sec}) \ln \left( \frac{60}{30} \right)}{\pi (81.6\text{ft}^2 - 77.7\text{ft}^2)}
\]

\[
k = 0.000426 \text{ ft/sec}
\]

\[
= 4.26 \times 10^{-4} \text{ ft/sec}
\]

**Given:**
- \( r_1 > r_2 \)
- \( r_1 = 60 \text{ ft} \)
- \( h_1 = 100' - 18.4' = 81.6' \)
- \( r_2 = 30 \text{ ft} \)
- \( h_2 = 100' - 22.3' = 77.7' \)

**References:** CERM 13 + 21-5 + Geotech Notes
Seepage and Flow Nets

The principles of seepage and flow nets apply when water flows through soil because of an imbalance in the total head (Second rule of civil engineering: water flows downhill). In other words, the movement of water occurs because hydrostatic conditions are not present and the traditional hydrostatic equations cannot be applied.

A flow net is a graphical representation of how water flows through a semi-permeable barrier such as an earthen levee or beneath an impermeable barrier such as a concrete or sheet pile dam. The purpose of the flow net is to allow for calculation of such information as; the total flow under or through the structure, the pore pressures at various points within or beneath the structure and the factor of safety against uplift failure.

Parts of a flow net

A flow net is composed of an intersecting grid of two types of curved lines; flow lines and equipotential lines. Flow lines represent the paths or channels that water will travel along as it flows through the soil. Equipotential lines join points of equal head within the soil and are analogous to contour lines on a topographical map. Rather than showing points of equal elevation, equipotential lines denote points of equal total head.

The area between two adjacent flow lines is known as a flow channel or a flow path.

The space between two adjacent equipotential lines is known as equipotential space and denotes a specific drop in head between the upstream and downstream line.

Constructing a flow net

The construction (drawing) of a flow net is a tedious and time consuming process. The following rules are meant to serve as a refresher and aid in the interpretation of a flow net that has already been drawn. In the event that an exam question requires the construction of a flow net (doubtful, but possible as an afternoon question), it is recommended that a predrawn flow net from
a reference source be modified to cover the given problem. In other words, if the problem deals with flow beneath a sheet pile dam, find a sheet pile dam flow net in a reference book and use it.

1. The flow net and impermeable structure are drawing to scale. This is needed so that the gradients can be calculated.
2. Each flow path or flow channel must carry the same amount of flow.
3. The difference in head between one pair of adjacent equipotential lines must be the same as for every other adjacent pair of equipotential lines.
4. Equipotential lines and flow lines must intersect at right angles.
5. The areas formed by the intersection of the equipotential lines and the flow lines should be roughly square.
   a. This does not mean that each square of the flow net will be of equal size, only that the center to center spacing between the adjacent flow lines should roughly equal the center to center spacing of the adjacent equipotential lines.
6. Spacing of the flow lines and the equipotential lines are inversely proportional to the seepage velocity, \( v_s \) and the hydraulic gradient.
   a. In general, flow lines and equipotential lines that are closer to the dam or drain will be closer together than flow lines and equipotential lines that are farther away.
   b. This means that flow channels will not be the same width from beginning to end.

Interpretation of flow nets

*Flow rate per unit length of structure*

This equation gives the flow per unit length of the structure, where unit length is measured in the same unit as head (i.e. feet, meters, etc). To find the total flow beneath the structure multiply the flow per unit length by the total length.

Pay attention to the time units on permeability and the time units that are asked for in the problems statement (days, weeks, months or years).
\[ q = k \left( \frac{n_f}{n_d} \right) \Delta H \]

Where:
- \( q \) = the flow beneath a unit width of the structure
- \( k \) = permeability
- \( n_f \) = number of flow channels in flow net
- \( n_d \) = number of equipotential drops in flow net
- \( \Delta H \) = difference in total head between headwater and tailwater

**Compute total head (h) at a point P**

The total head at a point within the flow net is affected by the fact that hydrostatic conditions do not exist because water is flowing.

\[ h = H_1 - n_{dp} \left( \frac{\Delta H}{n_d} \right) \]

Where
- \( H_1 \) = Total head at headwater
- \( n_{dp} \) = number of equipotential drops from headwater to point P
- \( n_d \) = total number of equipotential drops in flow net
- \( \Delta H \) = difference in total head between headwater and tailwater

**Pore pressure (u) at a point P**

\[ u = \gamma_w (h - z) \]

Where
- \( u \) = pore water pressure at point P
- \( h \) = total head at point P
- \( z \) = elevation head at point P

**Hydraulic gradient (i)**

\[ i = \frac{\Delta h}{\Delta L} \]

Where
- \( \Delta h \) = change in total head
- \( \Delta L \) = increment of length
Critical hydraulic gradient \((i_{cr})\)

Flowing water can cause soil particles to erode or lose a significant amount of shear strength. The more quickly head is lost (steeper the gradient) the greater the chance that there will be problems with strength loss. The critical hydraulic gradient is the ratio of the buoyant weight of the soil compared to the unit weight of water.

\[
i_{cr} = \frac{\gamma_b}{\gamma_w}
\]

Where

\[
\gamma_b = \text{buoyant unit weight of the soil} = \gamma - \gamma_w
\]

\[
\gamma_w = \text{unit weight of water}
\]

Factor of safety \((FS)\) against quick condition or heave

The phrase “quick condition” refers to soil that acts as if it were weightless due to the upward flow of water. This is where the term quicksand comes from. Quick conditions can develop in nature due to upward water movement caused by artesian wells. Quick conditions can also develop at the bottom of excavations made below the water table or within cofferdams. The dry (or tail water) side of levees or dams can also experience quick conditions due to flowing water.

\[
FS = \frac{i_{cr}}{i_E}
\]

Where

\[
i_{cr} = \text{critical gradient}
\]

\[
i_E = \text{exit gradient}
\]

Countering quick conditions

Relief wells can be installed on the downstream side of dams where water may seep to the surface. The water that collects in the wells can then be conveyed off site under controlled conditions.

A blanket of heavy, coarse-grained materials can be installed in areas that are susceptible to damage by seepage. The material needs to be coarse enough to be permeable without being so coarse that the underlying material migrates into the coarse material and clogs the pore spaces. The installation of a filter fabric can help alleviate this problem.
Exercise 14.

The soil in the sketch below has a permeability $k = 100 \text{ ft/yr}$ and a total unit weight $\gamma = 130 \text{ pcf}$. Compute the following:

a. The flow beneath the sheetpile wall.
b. The pore pressure $u$ at Point B.
c. The exit gradient $i_e$. Assume $\Delta h = 2.5 \text{ feet}$.
d. The critical gradient $i_c$.
e. The factor of safety $FS$ against a quick condition.

---

**Given:**

- Flow paths, $N_f = 5$
- Equipotential lines, $N_p = 9$
- $\Delta H = 12 - 3 = 9 \text{ ft}$
- $\frac{\text{Loss}}{\text{equ line}} = \frac{\Delta H}{N_p} = \frac{9}{9} = 1 \text{ ft/lin.}$
(a) Flow beneath sheetpile

\[ q = k \Delta H \left( \frac{N_f}{N_p} \right) = (100 \text{ ft/yr}) (9 \text{ ft}) \left( \frac{5}{9} \right) = 500 \text{ ft/yr} \]

(b) Pore pressure, \( u \) at pt B,

\[ u = y_w (h - z) \]

\[ h = \text{total h at Pt. P} = 7.5 \text{ ft} \]
\[ z = \text{elev. at Pt. P} = -22.0 \text{ ft} \]

\[ u = 62.4 \text{ lb/ft}^3 \left( 7.5 \text{ ft} - (-22.0 \text{ ft}) \right) = 1840.8 \text{ lb/ft}^2 \]

(c) exit gradient, i.e. (Reference TAMU Geotech video notes)

assume \( \Delta h \) (length of last square) = 2.5 ft

\[ i_e = \frac{1 \text{ ft}}{2.5 \text{ ft}} = 0.4 \text{ ft/ft} \]

(d) Critical Gradient, \( i_{cr} = \frac{y_b}{y_w} = y - y_w \)

\[ i_{cr} = \frac{(130 \text{pcf}) - (62.4 \text{ pcf})}{62.4 \text{ pcf}} = 1.08 \]

(e) Factor of Safety, \( FS = \frac{i_{cr}}{i_e} \)

\[ FS = \frac{1.08}{0.4} = 2.7 \]
**Total Stress versus Effective Stress**

Soil at a specific depth below the surface feels stress (force per unit area) due to the weight of the soil above it (overburden) and because of the addition of overlying loads. There are two types of stress that must be considered in geotechnical engineering problems; total stress and effective stress.

Total stress is found by summing the weight of all of the material above a unit area of soil (i.e. one square foot).

**Geostatic Total Stress** (horizontal ground surface, no surface loading):

\[
\sigma_v = \sum_i (\gamma_i) (t_i)
\]

Where

- \(\sigma_v\) = total vertical stress at a specific depth below the ground surface
- \(\gamma_i\) = unit weight of layer \(i\)
- \(t_i\) = thickness of layer \(i\)

Generalized soil profile for calculating total stress.
Effective Stress

Effective stress can not be measured, it must be calculated. Knowing the effective stress within a soil profile is important because it is changes in effective stress, not total stress, that cause soil consolidation. Effective stress is a result of the buoyant force of water acting upward on submerged soil. For soils that are above the water table total stress equals effective stress.

An analogy describing the difference between total and effective stress is as follows. A weight lifter performing a bench press without a spotter is responsible for lifting the entire weight. This is equivalent to total stress. If the weight lifter uses a spotter, then the weight lifter is responsible for lifting a smaller portion of the load because the spotter is helping. The reduced load that the weight lifter feels is equivalent to effective stress. The portion of the load that the spotter carries is equivalent to the buoyant force of the water.

Effective Stress Principle

\[ \sigma' = \sigma - u \]

Where 
\[ \sigma' = \text{effective stress} \]
\[ \sigma = \text{total stress} \]
\[ u = \text{pore pressure} \]

Hydrostatic Pore Pressure

\[ u = \gamma_w (z - D_w) \]

Where 
\[ u = \text{pore water pressure} \]
\[ \gamma_w = \text{unit weight of water} \]
\[ z = \text{depth to point in question} \]
\[ D_w = \text{depth to ground water table} \]
Buoyant Unit Weight

Soil that is below the water table is saturated and experiences an uplift or buoyant force because of the surrounding water.

\[ \gamma_b = \gamma - \gamma_w \]

Where
- \( \gamma_b \) = buoyant unit weight
- \( \gamma \) = total (saturated) unit weight of soil
- \( \gamma_w \) = unit weight of water
Exercise 15.

Compute the effective vertical stress at the mid-depth of the clay layer in the figure below.

\[
\sigma' = (2\text{ ft})(132 \text{ pcf}) + (3\text{ ft})(132 \text{ pcf} + 62.4 \text{ pcf}) + (5\text{ ft})(126 \text{ pcf} - 62.4 \text{ pcf}) =
\]

\[
= 264 \frac{\text{lb}}{\text{ft}^2} + 208 \frac{\text{lb}}{\text{ft}^2} + 318 \frac{\text{lb}}{\text{ft}^2}
\]

\[
\sigma' = 790 \frac{\text{lb}}{\text{ft}^2}
\]
Stress Distribution from Applied Loads

Many aspects of geotechnical engineering such as foundation design and consolidation theory depend upon the fact that the stress of applied loads on a soil mass dissipate with increasing depth. This dissipation occurs because more and more soil is mobilized with the soil mass to resist or support the applied load.

A “back of the envelope” type of calculation would be to assume that the area of soil that supports the applied load increases 1 foot horizontally for every 2 feet of depth within the soil mass. For example, if a 5 feet square foundation is constructed on the surface of the soil, then the area that supports the load at the surface in 25 square feet. Six feet beneath the surface of the soil, the size of the area that is supporting the load has increased to 11 feet by 11 feet (the initial 5 feet on each side, plus 3 feet on each of the 4 sides) or 121 square feet.
Elasticity solutions

The majority of the non-finite element solutions used to determine the increase in stress due to applied loads are based on elastic theory. This means that it is assumed that the loads do not cause permanent deformation of the soil. This assumption is not true, but it provides a basis for analysis.

Elastic solutions are not applicable to conditions at or near failure state because significant permanent deformations usually occur the closer the soil is to failure. This is one of the reasons that foundations are designed based on an allowable soil stress rather than an ultimate soil stress.

The change in vertical stress, $\Delta \sigma_v$ due to the applied load is assumed to be independent of the elastic modulus, $E$ and the Poisson’s ratio $\nu$.

The horizontal stress due to the applied load is dependent on the Poisson’s ratio $\nu$.

Poisson’s ratio compares horizontal strain to vertical strain:

$$\nu = \frac{\varepsilon_h}{\varepsilon_v}$$

*Chart solutions using Boussinesq theory*

A weakness of the 2:1 load dissipation solution is that the method does not account for the fact that pressure within the center of a loaded area is usually higher than the pressure at or near the edge. The variation in pressure occurs because the loaded area (foundation) is usually rigid and the center portion of the loaded area is more affected by the load from adjacent portions of the loaded area than the edges.

Various tables and charts have been developed to address the variation in pressure within a loaded area. The charts are used to find an influence factor, $I$ for a point of interest. The point of interest is specified by a ratio of the vertical and horizontal distance below and away from a given point within the loaded area to a dimension of the footing.

$$\Delta \sigma_v = p I$$
Where

\[ \Delta \sigma_v = \text{the CHANGE in vertical stress due to the applied load} \]
\[ p = \text{the uniform pressure applied by the load at the base of the footing} \]
\[ I = \text{the intensity factor (always} \leq 1) \text{at the point in question} \]

Remember, the above formula is used to find the increase in pressure due to the applied load. The total pressure is the sum of the pressure that existed prior to the load being applied and the change in pressure.

Important considerations for influence factor charts on the following pages:

Circular Loaded Area:

The ratios used to enter the chart are based on the radius of the loaded area, \( r \). To use the chart, the ratio of the distance of the point in question below the bottom of the loaded area, \( z \) is divided by the radius of the loaded area (i.e. \( z/r \)). Values for the \( z/r \) ratio are plotted on the sides of the chart. The curved, solid lines represent differing ratios of the horizontal distance of the point in question from the CENTER of the loaded area, \( x \) (i.e. \( x/r \)).

Once the two ratios are known, determination of the influence factor, \( I \) is simply a matter of finding out the point where the two ratios intersect. For example, if \( z/r = 2 \) and \( x/r = 5 \), then the value of \( I \) is about 0.003. This means that the increase in pressure at a point that is \( 2r \) below the loaded area and \( 5r \) from the center of the loaded area feels about a 0.3% increase in pressure due to the applied load.

Rectangular or Square Loaded Area:

The chart dealing with square and rectangular loaded areas is used to find the change in pressure beneath a CORNER of the loaded area. As such, attention must be paid to what point the problem statement asked for. If the question asks for the change in pressure beneath the center of the loaded area, then divide the loaded area into 4 equal sub-areas that have a corner directly above the point in question. The resultant influence factor is then multiplied by 4 to arrive at the change in stress. If the point in question is located at some other point, such as halfway between the center and one of the edges, then the loaded area will be sub-divided into 4 unequal areas, each with a
corner above the point in question. The final value of I will be the sum of each of the sub-area I’s.

The loaded area has dimensions of x and y. The depth below the loaded area of the point in question is designated as z. Two values, m and n are needed to enter the chart and find the influence factor I. M and n are ratios of the x and y dimensions and z (i.e. \( m = x/z \) and \( n = y/z \)).

In the chart, the horizontal axis shows values of n on a log scale. The contour lines are values of m. The values of I are located along the vertical axis. Assume we have a square footing that is 5 feet wide by 5 feet long and we are interested in the change in stress 5 feet beneath one of the corners. In this case \( n = m = 1 \) and \( I = 0.18 \). This means that the pressure due to the applied load has been reduce by about 88% by the time we are one unit depth beneath the corner of a square footing, regardless of the size of the footing.

*Important special case – infinite uniform load: \( I = 1.0 \)*

There is a point at which a load becomes so large that there is no reduction in pressure until depth becomes very large. For example, suppose that an international airport was to be built on an island that would be constructed just for this purpose. The island would be about 13,000 feet long by 4,000 feet wide. If the island was considered to be one large footing, then it becomes apparent that the soil within the first several hundred feet of the base of the island would generally feel the full loading imparted by the island.

*Superposition*

The principle of superposition is used to determine stresses resulting from complex loading conditions. The final loading can be computed by adding or subtracting solutions from simple loading geometries. For example, the affect of constructing multiple storage tanks near an existing building can be determined by analyzing the affect of each individual storage tank and then adding the individual results.
FIGURE 5
Influence Value for Vertical Stress Under Uniformly Loaded Circular Area (Boussinesq Case)
LOAD $p$ PER UNIT OF AREA

$\sigma_y = I \times p, m = \frac{x}{y}, n = \frac{y}{z}$

$m$ AND $n$ ARE INTERCHANGEABLE

**FIGURE 4**
Influence Value for Vertical Stress Beneath a Corner of a Uniformly Loaded Rectangular Area (Boussinesq Case)
Exercise 16.

A round bulk storage tank is to be built upon a sandy soil. The diameter of the tank is planned to be 80 feet and the uniform load caused by the tank and its contents at the soil surface has been calculated to be 2,000 psf.

Determine the increase in vertical stress caused by the surface loading of the tank and its contents at Points A, B and C using Boussinesq theory.

\[ \Delta \sigma_v = \rho \frac{r^2}{2} \]

\[ \rho = 2000 \text{ psf} \]

\[ r = 40 \text{ ft} \]
Solution for Pt. A

\[
\frac{z}{r} = \frac{40}{40} = 1 \quad \frac{x}{r} = \frac{0}{40} = 0
\]

From graph, \( I = 0.75 \) \( \Rightarrow \) \( \Delta \sigma_v = (2000 \text{ psf})(0.75) \)
\( \Delta \sigma_v = 1500 \text{ psf} \) \( \Rightarrow \) \( \sigma_v = 2000 + 1500 = 3500 \text{ psf} \)

Solution for Pt. B

\[
\frac{z}{r} = \frac{60}{40} = 1.5
\]
\[
\frac{x}{r} = \frac{40}{40} = 1
\]

From graph, \( I = 0.275 \)
\( \Rightarrow \) \( \Delta \sigma_v = (2000)(0.275) = 550 \text{ psf} \)
\( \Rightarrow \) \( \sigma_v = 2500 \text{ psf} \)

Solution for Pt. C

\[
\frac{z}{r} = \frac{20}{40} = 0.5 \quad \frac{x}{r} = \frac{60}{40} = 1.5
\]

From graph, \( I = 0.04 \)
\( \Rightarrow \) \( \Delta \sigma_v = (2000)(0.04) = 80 \text{ psf} \)
\( \Rightarrow \) \( \sigma_v = 2080 \text{ psf} \)

I used CERM contour chart easier to see
CERM (13): A = 83
Shear Strength

Shear strength is a measure of a material’s ability to resist sliding along its internal surfaces. It is the shear strength of a soil, rather than its compressive strength, that allows soil structures with sloping sides (dams, levees, embankments and natural slopes) to maintain their equilibrium. Shear strength also plays a large roll in bearing capacity of foundations and in lateral earth pressure.

There are several formulas to define the failure shear stress in soils. The Mohr-Coulomb strength envelope is one of the most common:

\[ \tau'_{f} = c' + \sigma' \tan \phi' \]

Where
- \( \tau'_{f} \) = effective shear stress at failure
- \( \sigma' \) = effective normal stress on the failure plane
- \( c' \) = effective cohesion
- \( \phi' \) = effective angle of internal friction

\[ \sigma' = (\sigma - u) \]

Where
- \( \sigma \) = normal stress on the shear plane
- \( u \) = the pore water pressure

Components of Shear Strength

As shown above, shear strength depends upon several factors; the effective normal stress on the failure plane, the effective cohesion of the soil and the effective angle of internal friction. The contribution of each of these factors can be described by the following analogy.

Suppose that a stack of papers was resting upon a smooth surface. The amount of laterally applied force needed to cause the papers to move would be the shear force. The magnitude of the shear force needed to cause the movement would not always be the same. If a weight was placed on top of the papers (normal force) the shear force needed to move the papers would increase. If a sticky substance was placed beneath the bottom most paper
(cohesion) the shear force would also change. An adhesive being placed between each of the pages (angle of internal friction) would also cause the magnitude of the shear force to increase.

As a practical manner, soil that is located further from the surface will generally have a higher shear strength due to the normal force (overburden pressure) being higher. This assumes that the soil in question is similar in all other properties.

**Determining Shear Strength**

Shear strength in soil is generally determined in one of two ways; the direct shear test or the triaxial test. The direct shear test is primarily used on remolded samples of coarse grained material and the triaxial test is generally used on fine grained soils.

**Direct Shear Test**

*Set up*

Operation

One half of the test device is stationary and the other half is free to move laterally. Force is applied to the free half until the sample fails in shear along a horizontal plane. The amount of normal (vertical) force can be varied to allow for multiple tests to be conducted on sand or gravel that has been compacted in the test chamber to the same unit weight.

The direct shear test for coarse grained soils is usually assumed to be a drained test because excess pore water pressure dissipates quickly from coarse grained soils.

Results

Each test produces one data point on a graph of effective shear stress at failure, \( \tau' \) versus the applied normal effective stress, \( \sigma' \). At least 3 tests (data points) should be conducted to obtain a reasonable result. A best-fit line is drawn through the data points. The y-intercept is the effective cohesion, c’. The angle that the line makes from the horizontal is the effective friction angle, \( \Phi' \).
Plot of the results of a direct shear test. Note that the scales on the x and y axis are the same.

Triaxial Shear Test

A drawback to the direct shear test is that the failure plane is forced to occur on a horizontal plane, which may not adequately model field conditions. The triaxial shear test does not force the failure plane to occur at a specific place. Additionally, the triaxial test can be conducted in a drained or undrained manner and it is possible to monitor the pore pressures that develop during testing. As such, an undrained test, which is faster than a drained test, can be conducted and the results used to obtain drained soil parameters.
Set up


Operation

The test sample is encased in a rubber membrane and is saturated. A porous stone on the bottom and/or top of the sample allows for drainage, if the release valve is open. The cell is pressurized if a confining pressure, $\sigma_3$ is to be used during the test. The axial compressive force, $P$ is applied at a constant rate until failure occurs. Failure can be defined as the sample shearing on a defined plane, as shown below, or by undergoing excessive deformation.
Clay soil sample that sheared on a fairly defined plane.

*Free body diagram of triaxial specimen*

A free body diagram of a triaxial test specimen is shown below. There 3 primary or principle stresses associated with the diagram:

\[ \sigma_1 = \text{the major principle stress} \]
\[ \text{aka: vertical stress or axial stress} \]
\[ \sigma_2 = \text{the intermediate principle stress} \]
\[ \sigma_3 = \text{the minor principle stress} \]
\[ \text{aka: confining pressure or cell pressure} \]

In this case \( \sigma_2 = \sigma_3 \) because the sample is loaded with the same pressure in the side to side plane as it is in the front to back plane.
The minor principle stress is also called the confining or cell pressure in triaxial tests because the test cell is pressurized to $\sigma_3$ in order to model the confining pressure that the soil sample experienced in the field. The entire sample feels the confining pressure. As a result:

$$\sigma_1 = \sigma_3 + \text{the axial pressure added during the portion of the test (P/A)}.$$  

Where  
$P$ = the applied load  
$A$ = the cross sectional area of the sample
Drained Shear versus Undrained Shear Strength

Soil that is below the water table will react differently to increases in stress than soil above the water table. The difference in behavior is related to how quickly increased pore water pressure, \( u \) is dissipated. Soils above the water table do not experience appreciable increases in pore water pressure compared to soils below the water table. Additionally, soils with high permeability, such as sands and gravels, can rapidly relieve increased pore water pressure compared to fine grain soils like silts and clays.

Slow draining, saturated soils initially feel the increase in applied load in the pore water. The load is gradually transferred to the soil grains as the pore water gradually leaves the loaded soil mass. Once all of the increase in pore water pressure has been dissipated, all of the new load is carried by the soil grains.

This concept is illustrated in the diagram below. The closed cylinder is filled with water and contains a spring. A plunger represents the applied load and an outlet valve represents the permeability of the soil. The greater the permeability, the faster the pore water can move out through the valve. When the soil is initially loaded all of the increase in load is carried by the water, which results in an increase in pore pressure. As water flows out of the valve the pore pressure decreases and load is transferred to the spring, which compresses. Eventually, all of the excess pore water pressure due to the applied load is relieved and the spring (soil particles) is carrying all of the applied load.
Which Shear Strength to use in Calculations

The decision of which shear strength value, drained or undrained, to use in design depends upon the conditions that will be present in the field.

Undrained Shear Strength

Undrained shear strength is generally used in the following conditions:

1. Fine-grained, saturated soils,
2. Soils that are not able to dissipate excess pore water pressure due to a lack of drainage, and
3. Soils that maintain a constant volume during loading.
Practical Situations:

The situations during which undrained shear strength may dominate design can occur during or immediately following construction because the excess pore water pressures have not had time to dissipate. This is especially true when loading occurs rapidly. What constitutes rapid loading depends greatly upon the permeability of the soil and how quickly the excess pore water pressure can dissipate.

Laboratory Evaluation

There are several tests that can be used to provide the shear strength of soil. Which test to use depends upon what conditions and loading are encountered in the field.

*Unconsolidated Undrained (UU) Triaxial Test*

A saturated, cylindrical sample of cohesive soil is placed in the test cell and the drainage valves are closed. The test cell is pressurized to a level that generally equals the in-situ pressure that the sample felt before it was removed from the field. Because the drainage valves are closed the sample, which probably relaxed or expanded when it was removed from the field, can not reconsolidate to its previous volume.

An axial stress is then applied to the sample until it fails. Failure can be defined as excessive deformation (the sample bulges in the middle but does not shear) or when shear occurs. Again, since the drain valves are closed the excess pore water pressure can not escape and the test is undrained.
Results

The distance between $\sigma_1$ and $\sigma_3$ is the diameter of the Mohr’s circle. The radius of the circle equals the undrained shear strength, $S_u$.

$$S_u = c = (\sigma_1 - \sigma_3)/2$$

*Consolidated Undrained (CU) Triaxial Test*

The CU test is similar to the UU test. The main difference is that the sample is allowed to consolidate once the confining pressure is applied. Consolidation is assumed to have finished when the excess pore water pressure resulting from setting the cell pressure has dissipated. Once the excess pore water has dissipated the drain valves are closed and the test continues as with the UU test.
Use of CU in practice

Rapid drawdown (drainage) of embankment dams, levees or canals.

Failure envelope for consolidated-undrained tests.

Special Cases when c = 0

\[
\frac{\sigma_1}{\sigma_3} = \tan^2 (45 + \frac{\Phi}{2})
\]

\[
\Phi = 2[(\tan^{-1}\left(\frac{\sigma_1}{\sigma_3}\right)^{1/2}) - 45^\circ]
\]

Steps to solve the \( \Phi \) equation:
1. take the square root of the ratio of major to minor principle stresses
2. take the arc tangent of the ratio found in step 1
3. subtract 45° from the answer found in step 2
4. multiply the answer from step 3 by 2
Unconfined Compression

As the name implies, the unconfined compression test does not apply a confining pressure to the test sample. The test is conducted in much the same way as a concrete compression test.

Use of unconfined compression test in practice

The unconfined compression test is rapid test for cohesive soils. The test provides reasonable results for soils that are close to the surface, where the confining pressure is minimal. A disadvantage of the test is that the lack of confining pressure can allow the soil to prematurely fail along planes of weakness such as sand seams, inclusions or slickensides.

Results

The undrained shear strength, $S_u$ is half of the unconfined compressive strength. The undrained shear strength also equals the cohesion, $c$. 
Unconfined compression test failure envelope.

Field Evaluation

The undrained shear strength can be evaluated by field methods such as:

- Cone Penetration Test
- Field Vane Test
Exercise 17.

A UU test is performed on saturated clay. The minor and major principal stresses on the specimen at failure are 2,000 psf and 3,500 psf respectively.

Determine the undrained shear strength of the specimen, $S_u$.

\[ S_u = c = \frac{\sigma_1 - \sigma_3}{2} \]

Minor stress, $\sigma_3 = 2000$ psf
Major stress, $\sigma_1 = 3500$ psf

\[ S_u = \frac{3500 - 2000}{2} = 750 \text{ psf} \]
Exercise 18.

A CD test is performed on an uncemented poorly graded sand. The confining pressure in the test cell is 1,000 psf. The measured major principal stress at failure is 3,300 psf.

Determine the effective angle of internal friction $\phi'$.

$$\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \quad \Rightarrow \quad \phi = \sin^{-1}\left(\frac{2300}{4300}\right) = 32.3^\circ$$

Since there is no excess pore pressure, total stresses equal effective stresses and $\phi = \phi' = 32.3^\circ$

Exercise 19.

A CU test is performed on an uncemented poorly graded soil (SP-SM). The total confining pressure in the test cell is 1,000 psf. The measured total vertical stress at failure is 1,700 psf, and the measured pore water pressure at failure is 650 psf.

Determine the effective angle of internal friction $\phi'$.

At failure, $\sigma_3 = 1000\text{psf}$

$$\sigma_{\text{Total}} = 1700\text{psf}$$

$$\sigma_1 = \sigma_3 + \sigma_{\text{fail}} = 2700\text{psf}$$

Effective stresses:

$$\sigma_3' = \sigma_3 - U_{\text{fail}} = 1000 - 650 = 350\text{psf}$$

$$\sigma_1' = \sigma_1 - U_{\text{fail}} = 2700 - 650 = 2050\text{psf}$$
Consolidated-undrained angle of shearing resistance, $\phi$

$$\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

Drained friction angle, $\phi'$

$$\sin \phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

$$\phi' = \sin^{-1} \left( \frac{2050 - 350}{2050 + 350} \right)$$

$$\phi' = 45^\circ$$
Lateral Earth Pressure

Unlike water, the horizontal pressure that soil exerts at a specific depth is not the same as the vertical pressure at the depth in question. The difference is primarily due to the fact that soil has shear strength and water does not.

Lateral earth pressure is calculated on an effective stress basis, which means that the unit weight of water, $\gamma_w$ must be subtracted from soil that is beneath the water table.

$$\sigma'_h = K \sigma'_v$$

Where

- $\sigma'_h$ = the effective horizontal earth pressure at a specific depth
- $K$ = earth pressure coefficient
- $\sigma'_v$ = the effective vertical stress at a specific point

Lateral earth pressure problems are usually solved by plotting the stress distribution curve along the structure (retaining wall, dam, pile etc) that the soil is action on and then determining the magnitude and position of the resultant forces. This procedure is very similar to the way that distributed load problems are solved in structures. Each soil layer will generally have its own value of $K$ because of differences in the effective angle of internal friction, $\phi'$.  

To get total lateral stress, $\sigma_h$, which is the value used in calculating driving and overturning forces, the affect of the water table (i.e. pore pressure) must be accounted for:

$$\sigma_h = \sigma'_h + u$$

Where

- $u$ = pore pressure
- $\sigma'_h$ = the effective horizontal earth pressure

The affect of the water table, where present, is best handled by treating the water as a separate entity. The stress distribution curve for the water table will be zero at the top of the water table and increase at a constant rate of 62.4 psf per foot of depth below the water table.
The magnitude of a triangular pressure distribution is \( \frac{1}{2} \text{(base)}(\text{height}) \). The resultant force of the pressure distribution acts at the third point from the base of the distribution curve. Complex geometries are best handled by superposition.

**Lateral Earth Pressure Coefficient (K)**

The value of K that is used in the lateral earth pressure formula depends upon how the soil is acting, or being acted upon. There are three options: \( K_0 \), \( K_a \) and \( K_p \)

*At rest earth pressure coefficient (\( K_0 \))*

\( K_0 \) conditions occur when there is no lateral strain. This means that the soil can increase or decrease in the vertical dimension, but that it will not increase or decrease in the lateral dimension. This condition could occur in a one dimensional settlement problem or it could be a design case were no lateral forces are allowed to bear on a retaining wall.

\[
K_0 = 1 - \sin \phi' \quad \text{(empirical for sand)}
\]

Where \( \phi' = \) the effective angle of internal friction of the soil

\[
K_0 = 0.19 + 0.233 \log (\PI) \quad \text{(normally consolidated clays)}
\]

*Active earth pressure coefficient (\( K_a \))*

Active earth pressure conditions occur when the soil is moving toward the object that is holding it back (retaining wall, dam, sheet pile etc.). Another way of describing what is occurring is to say that the soil is undergoing lateral extension.

\[
K_a = \tan^2 (45 - \phi'/2) = 1/\tan^2 (45 + \phi'/2) = 1/K_p
\]
Passive earth pressure coefficient ($K_p$)

Passive earth pressure conditions occur when the soil is being pushed on. This phenomenon can occur on the front face of a retaining wall that is being pushed on its back side by a taller amount of soil. Soil undergoing passive pressure experiences lateral compression.

$$K_p = \tan^2 \left(45 + \frac{\phi'}{2}\right) = \frac{1}{K_a}$$

Typical values of $K_a$ and $K_p$

The value of $K_a$ for angles of internal friction ranging from 0 to 50 degrees is 1 to 0.13. An average value for angles of internal friction ranging from 25 to 35 degrees is 1/3.

The value of $K_p$ for angles of internal friction ranging from 0 to 50 degrees is 1 to 7.55. An average value for angles of internal friction ranging from 25 to 35 degrees is 3.
Exercise 20.

For the basement wall shown in the figure below:

a. Plot the distribution of total active earth pressure acting on the wall.
b. Compute the active horizontal thrust acting on the wall due to the soil.

You may assume: (1) no interface friction between the soil and the wall, and 
(2) the total unit weight of the silty sand SM is the same above and below 
the groundwater table.

GP: \( c' = 0, \phi' = 40^\circ \)
\( \gamma = 125 \text{ pcf} \)

SM: \( c' = 0 \)
\( \phi' = 34^\circ \)
\( \gamma = 130 \text{ pcf} \)
Gravity Retaining Walls

Gravity retaining walls rely on the weight of the wall and the friction between the bottom of the wall and the soil to restrain the material behind the wall. The loading condition that results in the greatest driving forces usually occurs when the soil behind the wall is saturated. Be sure to consider all forces acting on the wall: active, passive, wall weight, and soil weight if the wall has a sloped (battered) rear wall instead of a vertical rear wall.

Some possible cross-sections of gravity retaining walls.

Gravity retaining walls can become unstable and fail in one or more of the following manners:

- overturning about the toe (bottom of the front wall),
- sliding,
- bearing capacity failure,
- settlement, and
- slope stability failure

Stability Checks:
- *Overturning about toe:* \( FS = \frac{\Sigma M_R}{\Sigma M_O} \)

  \( M_R = \text{moments that resist overturning} \)

  \( M_O = \text{moments that cause overturning} \)

  \( FS = \text{Factor of Safety against overturning} \)
- **Sliding**: $FS = \Sigma F_R / \Sigma F_D$

  $F_R =$ resisting forces

  $F_D =$ driving forces

  $FS =$ Factor of Safety against sliding

- **Bearing Capacity**: see bearing capacity under inclined, eccentric loads

- **Settlement**: see settlement calculations

- **Overall stability**: see slope stability

**Methods of Solution**

There are two main theories used to find the magnitude of the horizontal forces acting on retaining walls; Rankine and Coulomb.

Rankine theory assumes that the ground surface above the back of the retaining wall is flat, which means that there is no soil surcharge. Rankine theory also assumes that the back of the retaining wall is vertical (no batter) and that there is no friction between the soil and the vertical wall.

Coulomb theory is used when there is a sloped surcharge load behind the wall, there is a slope to the rear wall and/or friction between the rear wall and the soil is taken into account.

**Rankine Active**

Soil moves toward wall, horizontal ground, no soil-wall friction

$$\sigma'_{ha} = K_a \sigma'_{v} - 2 \ c \ (K_a)^{1/2}$$

Where  
$c =$ cohesion (sand generally is assumed to have zero cohesion)  
$K_a = \tan^2 (45 - \phi'/2) = 1/\tan^2 (45 + \phi'/2) = 1 / K_p$
Rankine Passive

Wall pushes into soil, horizontal ground, no soil-wall friction

\[ \sigma'_{hp} = K_p \sigma'_v + 2 c (K_p)^{1/2} \]

\[ K_p = \tan^2 (45 + \phi'/2) = 1/K_a \]

Coulomb Theory

\[ P_a = \frac{1}{2} K_a \gamma'H^2 - 2 c (K_a)^{1/2} \]

Where \( \gamma' \) = the effective unit weight of the soil
\( H \) = the height of the retaining wall
Pa acts 1/3 of the distance up from the base of the retaining wall. The angle that \( P_a \) makes with the horizontal is \( (\delta + \theta) \). This means that the magnitude of the horizontal force, \( P_{ah} \) will equal \( P_a(\cos(\delta + \theta)) \) and that the magnitude of the vertical force, \( P_{av} \) will equal \( P_a(\sin(\delta + \theta)) \).

\[
K_a = \frac{\cos^2(\phi - \theta)}{\cos^2 \theta \cos(\delta + \theta) \left[ 1 + \sqrt{\frac{\sin(\delta + \phi) \sin(\phi - \alpha)}{\cos(\delta + \theta) \cos(\theta - \alpha)}} \right]^2}
\]

Where
- \( \phi' = \) the effective angle of internal friction,
- \( \alpha = \) the friction angle between the back face of the retaining wall and the soil,
- \( \theta = \) the angle the soil surcharge makes with the horizontal,
- \( \delta = \) the angle the back face of the wall makes with the vertical

\[
P_p = \frac{1}{2} K_p \gamma' H^2 + 2 c (K_p)^{1/2}
\]

**Notes:**

It is very important that you are comfortable with your calculator when solving complex equations like the one needed to obtain the Coulomb \( K_a \).

Make sure you have your calculator set to degrees and not radians.

Uniform surcharge loads behind a foundation wall due to structures or soil with a flat profile \((\alpha=0)\) are accounted for in the following manner:

1. Multiply the surcharge load (normally denoted by a \( q \) term in psf) by the appropriate value of \( K \).
   a. Usually \( K_a \) if on the back of the retaining wall.
   b. Usually \( K_p \) if on the front side of the retaining wall.
2. The pressure distribution will be uniform (rectangular) within each soil layer (i.e. where each value of \( K \) is the same)
   a. The pressure distribution from a surcharge load is not affected by the water table.
3. The resultant of the pressure distribution is located at the center of the pressure distribution.

The soil pressure distribution diagram will shift to the left or the right where there are changes in K, such as at layer changes.

1. The value of the pressure distribution at the top of the new layer is found by multiplying the pressure at the bottom of the overlying layer by the K value of the new layer.
   a. A decrease in the K value will cause the pressure distribution diagram to shift left while an increase in K will cause the diagram to shift right.

Variations of the gravity retaining wall problem:

1. Determine the factor of safety against sliding and/or overturning for a retaining wall with given dimensions.
2. Find the width of retaining wall required to meet a specified factor of safety against overturning and/or sliding.

If not given, assume that the unity weight of concrete is 150 pcf.
Exercise 21.

Calculate the factors of safety against sliding and overturning for the gravity retaining wall shown below. The soil and concrete material properties are as follows:

Soil: Effective angle of internal friction = 32 degrees
      Cohesion = 0
      Total Unit Weight = 125 pounds per cubic foot

Concrete: Total Unit Weight = 150 pounds per cubic foot

You may neglect the effects of friction at the soil-wall interface for the earth pressure calculation. The soil-concrete friction angle at the base of the wall is \( \delta_{sc} = 20 \) degrees.
Exercise 21

Equations

\[ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \]

\[ P_a = \frac{1}{2} K_a y' H^2 - 2 e (K_a)^{1/2} \]

\[ F_{S\text{side}} = \frac{\sum W \tan \delta}{P_a} \]

\[ F_{S\text{rot}} = \frac{\sum M_B}{\sum M_0} \]

Solution

\[ P_a = \frac{1}{2} (0.31) (125 \text{pcf}) (10 \text{ft})^2 = 1937.5 \text{ lb/ft} \]

\[ \sum W = W_{\text{concrete}} = \left[10 \text{ft})(2 \text{ft}) + \frac{1}{2}(0 \text{ft})(10 \text{ft})\right] 150 \text{pcf} = 7500 \text{ lb} \]

\[ F_{S\text{side}} = \frac{7500 \tan 20}{1937.5} = 1.41 \quad F_{S\text{rot}} = \frac{39000}{6452} = 6.04 \]

\[ \bar{X} = \frac{\sum A_i x_i}{\sum A_i} = \frac{(20 \text{ft}^2)(7 \text{ft}) + (30 \text{ft}^2)(2/3)(6 \text{ft})}{50} \]

\[ \bar{X} = 5.2 \]

\[ G \sum M_{Pa} = -(1937.5)(3.33) = 6452 \]

\[ G \sum M_{Ra} = (7500)(5.2) = 39000 \]
Cantilever Retaining Walls

- Rely on flexural resistance of wall to resist active earth pressures
- Can use Rankine or Coulomb, Rankine often easiest
- See attached sketches for typical force assumptions
- Consider all forces: active, passive, wall weight, and soil weight

Stability Checks:
- **Overturning about toe:** \( FS = \frac{\sum M_R}{\sum M_O} \)
  
  \( M_R = \) moments that resist overturning
  
  \( M_O = \) moments that cause overturning

- **Sliding:** \( FS = \frac{\sum F_R}{\sum F_D} \)
  
  \( F_R = \) resisting forces
  
  \( F_D = \) driving forces

- **Bearing Capacity:** see bearing capacity under inclined, eccentric loads

- **Settlement:** see settlement calculations

- **Overall stability:** slope stability
Exercise 22.

For the cantilever retaining wall shown below, calculate the factor of safety against sliding.

\[ y_{\text{concrete}} = 150 \text{ pcf} \]

\[ \phi' = 34 \text{ degrees} \]
\[ C' = 0 \]
\[ \gamma = 132 \text{ pcf} \]

\[ \delta = 20 \text{ degrees} \]

\[ k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.283 \]

\[ P_p \times P_a = \frac{1}{2} k_a \delta' H^2 + 2 \varepsilon (k_a \delta')^2 \]

\[ F_{S_{\text{side}}} = \frac{2 W \tan \delta + P_p}{P_a} \]

\[ P_a = \frac{1}{2} (0.283)(132 \text{ pcf})(12 \text{ ft})^2 = 2690 \]

\[ P_p = \frac{1}{2} (0.283)(132 \text{ pcf})(1 \text{ ft})^2 = 19 \]

\[ W_1 = (1 \text{ ft})(12 \text{ ft})(150 \text{ pcf}) = 1800 \]

\[ W_2 = (1 \text{ ft})(10 \text{ ft})(150 \text{ pcf}) = 1500 \]

\[ W_{\text{sand}} = (2 \text{ ft})(2 \text{ ft})(132 \text{ pcf}) = 528 \]

\[ W_{\text{sand}} = (7 \text{ ft})(12 \text{ ft})(132 \text{ pcf}) = 11088 \]

\[ F_{S_{\text{side}}} = \frac{(14910) \tan 20 + 19}{2690} = 2.03 \]
**Braced Excavations**

Braced excavations can be thought of as parallel retaining walls. However, the presence of lateral bracing greatly alters the earth pressure distribution along the outside faces of the walls. The lateral earth pressure distribution depends upon the soil type and the stiffness of the soil.

Design issues:

*Strut loads*

The horizontal members between the sides of the excavation are called struts. It is a fairly common problem to be asked to determine the force in one or more of the struts.

Not every strut carries the same load. The magnitude of the load depends upon the tributary area that is supported by the strut. Tributary areas are calculated by taking the horizontal and vertical center-to-center spacing between the struts. The lower bound of the bottom most strut’s tributary area is halfway between bottom strut and the floor of the excavation.

![Side and top views of a braced excavation showing tributary areas.](image)
Additionally, the force in each strut depends only upon the loading applied on one side of the excavation. In other words, calculate the force based on the soil profile on one side of the excavation only. **DO NOT** calculate the force on one side of the excavation and double it. Think of the strut as a vertically loaded, ground floor column that has been rotated 90°. The compressive load in the column is the sum of the loads that are applied from above. The ground provides and equal and opposite reaction.

**Bending moments in wall**

Determination of the bending moments in the walls would require knowing the pressure distribution and the force in each strut. The wall could then be analyzed as a beam with a distributed load (the soil) with the struts serving as the hinged reaction points.

**Heave at base of excavation**

bearing capacity analysis

**Seepage**

Excavations that penetrate the ground water table will probably experience seepage into the excavation. The factor of safety against the base of the excavation experiencing a quick condition (aka boiling) should be evaluated by constructing a flow net and comparing the exit gradient to the critical gradient.
Soft clay: \( \frac{\gamma H}{c} > 6 \)

Stiff clay: \( \frac{\gamma H}{c} < 4 \)
Exercise 23.

A 20-ft deep cut is braced as shown in the figure below. The struts are spaced at 15-ft centers in plan. The sand has a total unit weight of 135 pcf and an internal friction angle of 33 degrees. Estimate the force in Strut 2.

\[
K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.29
\]

\[H = 20 \text{ ft}\]

\[\gamma = 135 \text{pcf}\]

\[
P_{\max} = 0.65 K_a \gamma H = 0.65 (0.29)(135 \text{pcf})(20 \text{ft})
\]

\[P_{\max} = 508.95 \text{ pcf}\]

\[
F_{\text{Strut } 2} = (508.95 \text{ pcf})(10 \text{ft})(15 \text{ft}) = 76,342.5 \text{ lb}
\]

\[
F_{\text{Strut } 2} \approx 76,343 \text{ lb}
\]
Anchors and Tiebacks

In certain construction situations there may not be enough room to construct a gravity or cantilever retaining wall of sufficient size to hold back the applied load. In these situations a wall with anchors or tiebacks is sometimes used. The purpose of the anchors or tiebacks is to resist the loads placed on the wall by the material behind the wall. The loads are resisted by the frictional forces that develop between the material behind the wall and the anchor or tiebacks. **The loads upon the wall are assumed to follow a Rankine/Coulomb active earth pressure distribution.**

There is an area behind the wall that is referred to as the active failure wedge. This failure wedge starts at the bottom of the inside face of the wall and extends at an angle of \((45 + \Phi/2)\) to top of the wall. It is within this area that cracks and fissures may develop as the soil in this area pushes out on the wall. **Tiebacks and anchors must extend beyond the soil within the failure wedge because the soil within the failure wedge can not be counted upon to provide any frictional resistance.**

Items to consider in tieback/anchor problems:
1. The frictional resistance developed by the anchor or tieback depends upon the normal force at the level of the tieback/anchor.
   a. Tiebacks deeper in the retained material will have a larger normal force than those higher in the retained material.
2. The frictional resistance also depends upon the surface area of the tieback or anchor.
   a. Tiebacks composed of flat straps develop frictional resistance on both sides of the straps.
   b. The surface area of grouted anchors should be used in the friction resistance calculations.
3. The load that each tieback or anchor will be required to hold back depends upon the tributary area of the wall that the tieback/anchor is attached to.
   a. The pressure at the vertical center of the tributary area can be used to determine the load on the tieback (load equals tributary area times the pressure on the area).
   b. The lateral loads imparted by a water table, if present, should be handled as with retaining walls (i.e. treat as a separate load).
Schematic of the failure wedge behind a wall. The angle of $45 + \Phi/2$ is measured from the horizontal.
Exercise 24.

A reinforced earth retaining wall system is comprised of concrete panels 3 feet high by 3 feet wide (into the page) that are restrained by a series of tiebacks embedded into the backfill.

a. How much force (in lbs) must be mobilized in Tieback #2 to hold Panel #2 in place?

b. If the tiebacks are comprised of 1-ft wide strips with a soil-tieback interface friction angle $\delta_{int} = 22$ degrees, how long must tieback #2 be to achieve a factor of safety of 3 against pullout?

\[ H = 9 \text{ ft} \]
\[ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.27 \]

\[ p_{max} = 0.65 \cdot K_a \cdot \gamma \cdot H = (0.65)(0.27)(132 \text{pcf})(9 \text{ ft}) = 208.5 \text{ lb/ft}^2 \]

\[ F_{\text{Panel 2}} = (208.5 \text{ psf})(3 \text{ ft})(3 \text{ ft}) = 1876.5 \text{ lb} \]

\[ F_{\text{Side}} = \frac{W \cdot \tan \delta}{F_{\text{Panel 2}}} \implies \text{Length of tieback} = \frac{(F_{\text{Panel 2}})(F_{\text{Side}})}{(\text{Width of tieback}) \cdot \tan \delta} \]

\[ \text{Length} = \frac{(1876.5 \text{ lb})(3 \text{ ft})}{(6 \text{ ft})(132 \text{ pcf}) \cdot \tan 22^\circ} = 17.6 \text{ ft} \approx 18 \text{ ft} \]
Slope Stability

Soil slopes become unstable when the shear stress applied along a surface within a slope becomes greater than the shear strength of the soil along that surface. The surface can be roughly linear in shape if there is a defined, linear boundary between two soil layers. This situation could occur if a roadway embankment is constructed over an existing sloping, impermeable layer.

Soil overlying a layer of impermeable material

There are some situations where one of a slope’s potential failure planes is fairly well defined. One case is when the slope overlies a sloping impermeable layer. The soil above the impermeable layer can easily fail along the interface.

![Soil layer overlying a sloping, impermeable layer.](image)

The notation in the above diagram is as follows:

- \( H \) = the vertical distance from the top of the slope to the bottom of the slope
- \( D \) = the vertical distance from the top of the slope to the start of the impermeable layer
- \( h \) = the maximum height of the top soil mass perpendicular to the impermeable layer
\[ h = \frac{D}{\sin \beta} (\sin \beta - \alpha) \]

\( \beta \) = the angle of the impermeable layer at the toe of the slope
\( \alpha \) = the angle of the top soil layer at the intersection of the impermeable layer
\( L \) = the contact distance between the top soil layer and the impermeable layer

The following quantities are needed to calculate the factor of safety against sliding between the 2 layers:

\[ W = \frac{Lh\gamma}{2} \]

Where
\( W \) = the weight of the soil in the top soil layer
\( \gamma \) = the unit weight of the soil layer
\( L \) and \( h \) as defined above

\( F_s = W(\sin \alpha) \)

Where \( F_s \) = the magnitude of the force trying to cause sliding

\( R_s = cL + W(\cos \alpha)(\tan \delta) \)

Where
\( R_s \) = the magnitude of the force trying to resist sliding
\( \delta \) = the angle of friction between the top and impermeable layers
\( c \) = the cohesion of the impermeable soil layer

The factor of safety against sliding is equal to:

\[ F_{S, \text{sliding}} = \frac{R_s}{F_s} \]

**Slope failure in cohesionless soil (c = 0 and \( \Phi > 0 \))**

Sand that is sloped at an angle from the horizontal (\( \beta \)) greater than its angle of internal friction (\( \Phi \)) will fail by sliding. The factor of safety of sloped sand against sliding is:

\[ F_{S, \text{sliding}} = \tan \Phi / \tan \beta \]
Exercise 25.

A 14 foot deep cut is to be made in an area with 2 distinct soil layers. Determine the factor of safety against sliding ($F_{S\text{slide}}$) at the interface between the permeable and highly impermeable layers if the final soil profile will be as shown below.

\[ F_{S\text{slide}} = \frac{R_s}{F_S} \]

\[ R_s = cL + W(\cos \alpha)(\tan \delta) \]
\[ F_S = W(\sin \alpha) \]
\[ W = Lh \gamma \quad L = \frac{D}{\sin \alpha} \]
\[ h = \frac{D}{\sin \beta} (\sin \beta - \alpha) \]

**Equations**

**Solution**

\[ h = \frac{8\text{ft}}{\sin 40^\circ} (\sin 40^\circ - 25) = 3.22\text{ft} \]
\[ L = \frac{8\text{ft}}{\sin 25^\circ} = 18.9\text{ft} \]
\[ W = (18.9\text{ft})(3.22\text{ft})(101\text{pcf}) = 3078 \frac{\text{lb}}{\text{ft}} \]
\[ F_S = (3078 \frac{\text{lb}}{\text{ft}})(\sin 25^\circ) = 1300 \frac{\text{lb}}{\text{ft}} \]
\[ R_s = 165 \frac{\text{psf}}{\text{ft}}(18.9\text{ft}) + (3078 \frac{\text{lb}}{\text{ft}})(\cos 25^\circ)(\tan 25^\circ) \]
\[ R_s = 3118.5 \frac{\text{lb}}{\text{ft}} + 1127.1 \frac{\text{lb}}{\text{ft}} = 4245.6 \frac{\text{lb}}{\text{ft}} \]

\[ F_{S\text{slide}} = \frac{4245.6 \frac{\text{lb}}{\text{ft}}}{1300} = 3.27 \]
Slope failure in cohesive soils ($c > 0$, and $\phi \geq 0$)

There are several methods that can be used to determine the stability of slopes in cohesive soils. The least time consuming method makes use of a stability number and 2 charts. Which chart to use depends upon whether or not the cohesive soil has an angle of internal friction, $\phi$ equal to 0 or greater than 0.

It is generally assume that saturated clay has an angle of internal friction, $\phi = 0$.

Types of slope failure

There are three general ways that slopes fail based on where the failure exits the slope: the toe, the base or the slope. Diagrams of each failure are shown below.

The following notation is used in each of the failure diagrams:

$H$ = the vertical distance from the top of the slope to the bottom (base of the slope,
$D$ = the vertical distance from the bottom of the slope to the top of an underlying stiff layer. It is assumed that the slope failure will not extend into the stiff layer,
$\beta$ = the angle that the bottom (aka toe) of the slope makes with the horizontal

Schematic of a toe failure.
Schematic of a base failure.

Schematic of a slope failure.

*Slope stability charts*

The 2 slope stability charts provided below should be sufficient to handle most slope failure problems relating to cohesive (clay) soils.

**Chart Notation**

The following notation is used in both of the slope stability charts shown above:

$$N_s = \text{the slope stability number} = \frac{(\gamma')(H)}{c}$$
Where $\gamma' = \text{the effective unit weight of the failing soil, if the soil is above the water table then use } \gamma \text{ (total unit weight)}$

$H = \text{the vertical height of the slope as shown on the slope failure diagrams above}$

$c = \text{the cohesion of the failing soil}$

The height of the slope is related to the depth to the stiff or unyielding layer by a factor called $n_d$.

$$n_d = \frac{H + D}{H} \quad \text{(see slope failure diagrams above)}$$

Where

$H = \text{the vertical height of the slope}$

$D = \text{the vertical distance from the bottom of the slope to the unyielding layer}$

$\Phi = \text{the angle of internal friction of the failing soil.}$

**Using the charts**

Problems involving slope stability can involve analyzing an existing slope where $c, H, \beta, \gamma'$, and $\Phi$ are given to determine if the required factor of safety has been satisfied.

Problems can also involve determining the maximum angle of the slope ($\beta$), or the height of the slope ($H$) that will still meet the required factor of safety.

**Special Conditions**

If $\beta$ is greater than 53° then the failure surface will always be a toe circle.

If $n_d$ is greater than 4 then the failure surface will always be a base circle.
Exercise 26.

A channel is to be excavated in the soil profile shown below. The main body of the channel must be 40 feet deep. It is 80 feet from the top of the soil to stiff layer. What is the maximum angle ($\beta$) that the sides can be sloped to and still maintain a factor of safety against sliding of 1.3?

\[
F_{\text{cohesive}} = \frac{N_0 c}{\gamma_{\text{eff}} H} \quad \Rightarrow \quad N_0 = \left( \frac{F_{\text{cohesive}}(\gamma_{\text{eff}}) H}{c} \right)
\]

Assume soil is not saturated, so $d'=0$.

\[
N_0 = (1.3)(117 \text{pcf})(40) = 7.6 \frac{N_0 = (1.3)(117 \text{pcf})(40)}{800 \text{ psf}} = 7.6 \quad \text{and} \quad d = \frac{D}{H} = \frac{40}{40} = 1
\]

Chart, CERM(13) Page 40-7, Figure 40-6

\[
\beta = 12.5^\circ
\]
Consolidation of Saturated Clay

Concepts

- Total consolidation has 3 components: immediate consolidation, primary consolidation and secondary consolidation
  - Immediate consolidation occurs almost as soon as the load is applied and is due to rearrangement of loose soil particles,
  - Primary consolidation occurs as water is expelled from voids due to an increase in effective stress,
    - Effective stress can increase due to the application of a load or by a decrease in the height of the ground water table.
  - Secondary consolidation occurs after the increase in effective stress has dissipated and is due to additional, long-term rearrangement of soil particles.
    - Some degree of secondary consolidation occurs while primary consolidation is occurring, but they are modeled as 2 separate entities.
- Applies to saturated clays,
  - Unsaturated soils are difficult to model and are often assumed to be saturated. Unsaturated soils will generally consolidate faster than unsaturated soils.
- Need to estimate magnitude of settlement and how long it takes,
  - These are 2 independent calculations.
- Parameters controlling magnitude of settlement: $C_c$, $C_s$, $\sigma'_p$
  - These parameters are the result of a laboratory consolidation test which leads to the development of an e-lop P curve.
- Parameters controlling time rate of settlement: $c_v$, $C_\alpha$
  - These parameters are also the result of laboratory tests
- All stress calculations are made in relation to the stresses at the center of the consolidating layer, which provides an average of the stresses at the top and bottom of the consolidating layer.
The $e - \log \sigma'_v$ or $e - \log P$ curve

- semi-log plot of void ratio, $e$ versus vertical effective stress, $\sigma'_v$ or $P$
- Compression index, $C_c$: slope of virgin compression curve
- Swell index, $C_s$: slope of unload-reload curve
  - aka: $C_r$
- Preconsolidation stress, $\sigma'_p$
  - Highest stress that the soil has seen in the past
    - Can be higher than the current stress if soil has been excavated or removed by erosion or if the area previously contained glaciers
- Normally consolidated: $\sigma'_{v0} = \sigma'_p$
  - The stress that the soil feels now is the highest stress that it has ever felt
- The e-log P curve will be a relatively straight line
- Overconsolidated: \( \sigma'_{v0} < \sigma'_{p} \)
  - The stress that the soil is feeling before new load is added is less than the maximum stress that the soil has felt in the past
- Overconsolidation ratio: OCR = \( (\sigma'_{p} / \sigma'_{v0}) \)

**Primary Consolidation Settlement Equation**

The general equation for primary settlement (i.e. the equation that works regardless of whether or not the final stress at the center of the layer is greater than or equal to the preconsolidation pressure) is as follows:

\[
S_c = \left[ \frac{C_s}{1+e_0} \right] \log \left( \frac{\sigma'_{p}}{\sigma'_{v0}} \right) + \left[ \frac{C_c}{1+e_0} \right] \log \left( \frac{\sigma'_{f}}{\sigma'_{p}} \right)
\]

Where
- \( H \) = the thickness of the consolidating layer
- \( e_0 \) = the in-situ void ratio before any additional load is applied
- \( \sigma'_{v0} \) = the initial vertical effective stress at the center of the consolidating layer before any additional load is added (i.e. the effective weight of the soil above the center of the layer)
- \( \sigma'_{f} \) = final stress at the center of the consolidating layer after the load has been applied

This equation will give the magnitude of the settlement in the units that were used for layer thickness, \( H \). Be sure to answer the question in the units that were asked for.
Exercise 27.

A 5-ft thick normally consolidated clay layer has the following properties:

Compression Index: \( C_c = 0.45 \)
In Situ Void Ratio: \( e_0 = 0.92 \)
In Situ Overburden Stress: \( \sigma'_{vo} = 1,500 \text{ psf} \)

An additional vertical stress of 1,500 psf is applied to the clay layer. Estimate the primary consolidation settlement.

\[
S'_{primary} = \frac{H}{1 + e_0} \log \frac{\sigma'_f}{\sigma'_p} = \frac{H C_c}{1 + e_0} \log \frac{\sigma'_f}{\sigma'_{vo}}
\]

\[
\sigma'_f = 1500 \text{ psf} + 1500 \text{ psf} = 3000 \text{ psf}
\]

\[
\sigma'_{vo} = \sigma'_p \quad \text{for normally consolidated}
\]

\[
S'_{primary} = \frac{(5\text{ ft})(0.45)}{1 + 0.92} \log \left( \frac{3000}{1500} \right) = 0.35 \text{ ft}
\]
Exercise 28.

A 5-ft thick over-consolidated clay layer has the following properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression Index:</td>
<td>$C_c = 0.45$</td>
</tr>
<tr>
<td>Swell-Reload Index:</td>
<td>$C_s = 0.07$</td>
</tr>
<tr>
<td>In Situ Void Ratio:</td>
<td>$e_0 = 0.92$</td>
</tr>
<tr>
<td>In Situ Overburden Stress:</td>
<td>$\sigma'_{vo} = 1,500$ psf</td>
</tr>
<tr>
<td>Preconsolidation Stress:</td>
<td>$\sigma'_p = 2,500$ psf</td>
</tr>
</tbody>
</table>

An additional vertical stress of 1,500 psf is applied to the clay layer. Estimate the primary consolidation settlement.

\[
S_c = \frac{C_s H}{1 + e_0} \log \left( \frac{\sigma'_p}{\sigma'_{vo}} \right) + \frac{C_c H}{1 + e_0} \log \left( \frac{\sigma'_v}{\sigma'_p} \right)
\]

\[
\sigma'_v = 1500 + 1500 = 3000 \text{ psf}
\]

\[
S_c = (0.07)(5 \text{ ft}) \log \left( \frac{2500}{1500} \right) + (0.45)(5 \text{ ft}) \log \left( \frac{3000}{2500} \right)
\]

\[
S_c = 0.04 + 0.09 = 0.13 \text{ ft}
\]
Time of Consolidation based on Terzaghi consolidation theory

Calculating the magnitude or amount of consolidation due to an applied load is only one-half of the problem. The time for that consolidation to occur is also of interest.

When a load is applied to a saturated clay the pore pressures in the clay increase by the amount of the applied load felt at the center of the clay layer. Remember, we are basing all of our calculations on what the center of the clay layer is feeling because it is an average of the stress between the top and bottom of the layer. At the start of loading all of the excess pressure is felt by the pore water and not the soil matrix. Over time the excess pore water pressure dissipates and the load is applied to the soil grains. Consolidation occurs as the pressure is transferred to the soil grains. The time for consolidation to occur depends upon how quickly the excess pore water pressure dissipates.

Terzaghi consolidation theory makes use of a dimensionless time factor, $T_v$ to estimate the time to reach a specific degree of consolidation (i.e. 50% of consolidation):

$$T_v = \frac{c_v}{H_{dr}} t / (H_{dr})^2$$

Where $c_v =$ Coefficient of consolidation (from a lab test)
$H_{dr} =$ length of drainage path (single or double)
$t =$ the time to reach the degree of consolidation for a value of $T_v$

The degree of consolidation, $U_z$ refers to what percentage of the consolidation has occurred.

$$U_z = \frac{(u_0 - u)}{u_0}$$

Where $u_0 =$ initial excess pore pressure immediately after load is applied
$u =$ excess pore pressure at time $t$

The time factor for a given amount of consolidation is constant and are shown in the table below.
Fig. 9.3 Consolidation for any location and time factor in a doubly
drained layer (after Taylor, 1948).

From: An Introduction to Geotechnical Engineering, Robert Holtz and
### Average Degree of Consolidation U versus Dimensionless Time Factor $T_v$

<table>
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<th>$U$ (%)</th>
<th>$T_v$</th>
<th>$U$ (%)</th>
<th>$T_v$</th>
<th>$U$ (%)</th>
<th>$T_v$</th>
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<td>80</td>
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</tr>
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<td>47</td>
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<td>81</td>
<td>0.588</td>
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**Solving for the time of consolidation, t**

Time of consolidation problems are usually concerned with finding out how long it will take for a percentage of the calculated consolidation to occur. The problem is addressed by rearranging the equation for $T_v$ shown above to solve for $t$.

$$t = \frac{(T_v)(H_{dr})^2}{C_v}$$

Where
- $H_{dr}$ = the drainage distance (units of length)
- $C_v$ = coefficient of consolidation (units of area per time)

Make sure the drainage distance is expressed in the same units as $C_v$ (i.e. if $C_v$ is in inches$^2$ per minute then $H_{dr}$ should also be expressed in inches, rather than feet).

Also do not forget the $H_{dr}$ is squared.

The amount of consolidation is not required to determine the time for consolidation to occur.

**Drainage distance $H_{dr}$**

The drainage distance is the longest distance that a particle of water would have to travel in order to leave the consolidating layer. The consolidating layer can be considered to be either singly drained or doubly drained. In a singly drained layer the water can exit through either the top or the bottom of the layer, but not both ($H_{dr} = H$). Doubly drained layers drain water from both the top and bottom of the layer ($H_{dr} = H/2$).
Schematic of a singly drained clay layer.

Schematic of a doubly drained clay layer.

Ways to decrease the time for consolidation to occur

1. Decrease the drainage distance by installing vertical drains through the consolidating clay layer.
   a. A drain can be as simple as a hole filled with sand or a manufactured wick drain.
   b. The drainage distance is decreased to the center to center spacing of the drains.
2. Surcharge the surface overlying the consolidating layer.
a. The heavier the surcharge the faster the consolidation process will be.
b. Construction can not begin until the surcharge is removed.
c. Surcharge in combination of drains will increase the rate of consolidation even more.
Exercise 29.

Determine the time required for 75% primary consolidation settlement to occur for the clay stratum in the figure below.

\[
\frac{\Sigma \text{equations}}{\text{for} \ \gamma} = \frac{(T_v)(H_{dr})^2}{C_v}
\]

At \( U = 75\% \), \( T_v = 0.477 \)

Doubly drained \( \Rightarrow \)
\( H_{dr} = \frac{H}{2} \)

\[
t = \frac{(0.477)(5 \text{ ft})^2}{35 \text{ ft}^2/\text{yr}} = 0.34 \text{ yr} \approx 4 \text{ months}
\]
Clay Foundations: Secondary Compression (Consolidation) Settlement

Secondary consolidation occurs because particles within the clay layer are rearranged due to the applied load. Theoretically, secondary consolidation occurs after all of the excess pore water pressure due to the applied load has dissipated. In reality secondary compression occurs along with primary consolidation.

\[ S_s = \left[ C_\alpha \frac{H}{1+e_p} \right] \log \left( \frac{t}{t_p} \right) \]

Where

- \( S_s \) = secondary consolidation settlement
- \( C_\alpha \) = coefficient of secondary consolidation
- \( H \) = initial layer thickness before any consolidation occurs (use full layer regardless of single or double drainage)
- \( e_p \) = void ratio at end of primary consolidation
- \( t \) = time after start of primary consolidation that amount of secondary consolidation is wanted
- \( t_p \) = time to end of primary consolidation = \( (H_{dr})^2/c_v \)
Exercise 30.

The deposit below is subjected to an additional applied load of 1,500psf. Estimate the secondary compression settlement after 20 yrs in the clay layer below if the coefficient of secondary compression is 0.02 and the void ratio at the end of primary consolidation $e_p$ is 0.85.

\[
\begin{array}{c|c|c}
2 \text{ ft} & \text{GWT} & 5 \text{ ft} \\
\hline
5 \text{ ft} & \text{Sand, SP } \gamma = 132 \text{ pcf} & \\
\hline
10 \text{ ft} & \text{Clay, CL } \gamma = 126 \text{ pcf} & C_v = 35 \text{ ft}^2/\text{yr} \\
\hline
& \text{Sand, SW } \gamma = 137 \text{ pcf} & \\
\end{array}
\]

Equations

\[
S_s = \frac{C_v}{\alpha} \frac{H}{1 + e_p} \log \frac{t}{t_p}
\]

where, $t_p = (H_{dr})^2 \frac{C_v}{C_v}$

and $H_{dr} = \frac{H}{2}$

Solution

\[
t_p = \frac{(2.5 \text{ ft})^2}{35 \text{ ft}^2/\text{yr}} = 0.179 \text{ yr}
\]

\[
S_s = \frac{(0.02)(10 \text{ ft})}{1 + 0.85} \log \frac{20 - 0.179}{0.179}
\]

\[
S_s = 0.25 \text{ ft}
\]
Settlement in foundations built on sand

Settlement calculations of foundations built on sand are generally solved based on empirical methods and field tests. This is because it is practically impossible to obtain and transport undisturbed samples of sand to the lab and conduct meaningful tests.

Empirical Methods

- Based on penetration resistance (SPT N or CPT q_c)
- Net allowable pressure to limit settlement to 1 inch (typically)
- \( q_{\text{net(all)}} = q_{\text{f(all)}} - \gamma' D_f \)
  - \( \gamma' \) = the effective unit weight of the soil above the bottom of the footing
  - \( D_f \) = depth from the top of the soil to the bottom of the footing
- Relations vary: examples
  \[
  q_{\text{net(all)}} \text{ (ksf)} = \left( \frac{N_1}{2.5} \right) F_d S_e \quad \text{ (B < 4ft)}
  \]
  \[
  q_{\text{net(all)}} \text{ (ksf)} = \left( \frac{N_1}{4} \right) \left[ \frac{(B + 1)}{B} \right]^2 F_d S_e \quad \text{ (B > 4ft)}
  \]

- \( N_1 \) = the corrected SPT blow count at the depth of the bottom of the footing (correction adjusts for overburden effects)
- \( S_e \) = tolerable settlement in inches
- \( F_d \) = depth factor = \( 1 + 0.33 \) \( (D_f/B) \)
- \( B \) = the width of the footing (width is the smallest dimension of the contact area)

Plate Load Tests

Plate load tests are conducted by pressing a metal plate into the soil with the same contact pressure that is anticipated in the final footing. The test plates are generally much smaller than the final size of the footings. Plates that are too small may provide inaccurate data. The results of the load tests are “scaled up” by the following formula:

\[
S_F = S_P \left[ \frac{2B_F}{(B_F + B_P)} \right]^2
\]
Where

\[ S_F = \text{predicted settlement of footing} \]
\[ S_P = \text{measured test plate settlement} \]
\[ B_F = \text{footing width} \]
\[ B_P = \text{test plate width} \]
**Bearing Capacity**

There is a limit to the amount of weight that a soil can carry due to applied loads without failing. That failure limit is known as a soil’s ultimate bearing capacity. Foundations are not designed on the basis of a soil’s ultimate bearing capacity because the closer a soil gets to its ultimate capacity the greater the amount of consolidation and settlement.

**Allowable Bearing Pressure**

\[ q_a = \frac{q_{ult}}{FS} \]

Where 
- \( q_a \) = the allowable bearing capacity
- \( q_{ult} \) = the ultimate bearing capacity
- \( FS \) = the factor of safety against bearing capacity failure

**Net Bearing Pressure**

The above equation for allowable bearing capacity does not differentiate on how the soil is loaded. If the footing excavation is backfilled the bearing capacity must be reduced by the pressure caused by the backfill. The resulting bearing capacity is known as a net ultimate bearing capacity or a net allowable bearing capacity.

\[ q_{ult(\text{net})} = q_{ult} - q \]
\[ q_{a(\text{net})} = \frac{q_{ult(\text{net})}}{FS} \]

Where 
- \( q_{ult(\text{net})} \) = the net ultimate bearing capacity
- \( q_{ult} \) = the ultimate bearing capacity
- \( q \) = the pressure applied by backfilling the footing excavation
- \( q_{a(\text{net})} \) = the net allowable bearing capacity (aka design bearing capacity)

\[ q = \gamma'_1 D_f \]

Where 
- \( \gamma'_1 \) = the effective unit weight of the soil above the bottom of the footing
- \( D_f \) = the distance from the ground surface to the bottom of the footing
It is usually assumed that the unit weight of the soil above the footing is close enough to the unit weight of the footing material (concrete) that appreciable error will not be introduced into the calculations.

**General Bearing Capacity Equation**
Meyerhof’s Equations (other solutions: Terzaghi, Hansen)

\[
q_{ult} = c \cdot N_c \cdot s \cdot i \cdot d + \gamma'_1 \cdot D_f \cdot N_q \cdot s_q \cdot i_q \cdot d_q + \frac{1}{2} B \cdot \gamma'_2 \cdot N_{\gamma} \cdot s_{\gamma} \cdot i_{\gamma} \cdot d_{\gamma}
\]

Bearing capacity notation.

\( q_{ult} \) = ultimate bearing capacity (units of stress)
\( N_c, N_q, N_{\gamma} \) = bearing capacity factors
\( c \) = cohesion
\( s \) = shape factors
\( i \) = load inclination factors
\( d \) = depth factors
\( \gamma'_1 \) = effective unit weight of the soil above the bottom of the footing
\( D_f \) = depth from the ground surface to the bottom of the footing
\( B \) = footing width (smallest dimension of the footing contact area)
\( \gamma'_2 \) = effective unit weight of the soil below the bottom of the footing
Shape Factors: L = length of footing into page,

The basic bearing capacity equation was derived for a strip footing, which is considered to be infinitely long. In practical terms, the length of a strip footing is 10 times greater than the width (i.e. $L \geq 10B$).

For strip footing (infinitely long) all shape factors = 1.0

For rectangular footings:

$s_c = 1 + \frac{(B/L)(N_q/N_c)}{s_r = 1 - 0.4(B/L)}$
$s_q = 1 + \frac{(B/L)\tan \phi}$

For square and circular footings $B = L$ so:

$s_c = 1 + \left(\frac{N_q}{N_c}\right)$
$s_r = 0.6$
$s_q = 1 + \tan \phi$

Inclined Loading: $\alpha$ = angle of applied load measured from vertical

For vertical loads, all inclined load factors = 1.0

$i_c = [1-\left(\frac{\alpha}{90}\right)^2]$
$i_q = [1-\left(\frac{\alpha}{90}\right)^2]$

$i_r = 1$ if $\phi = 0$
$i_r = [1-\left(\frac{\alpha}{\phi}\right)^2]$ if $\phi > 0$: 


Schematic showing how $\alpha$ is measured.

Embedment Correction:

In general, the deeper the footing is below the ground surface the higher the bearing capacity due to the effects of the overburden.

$$d_c = 1 + 0.2 \sqrt{K_p}D_f/B$$

$$d_q = d_r = 1 \text{ if } \phi = 0$$

$$d_q = d_r = 1 + 0.1 \sqrt{K_p} (D_f/B) \text{ if } \phi > 0$$

$$K_p = \tan^2(45 + \phi/2)$$

$$\sqrt{K_p} = \tan (45 + \phi/2)$$

$D_f =$ depth of footing

Special Case when $\phi = 0$

$$K_p = \sqrt{K_p} = 1$$
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Modification for Eccentricity, \( e \)

An eccentric load condition occurs when the load is not applied to the center of the footing. One way that eccentric loadings are addressed is to reduce the area of the footing that is available to carry load. Bear in mind that the actual size of the footing is not reduced, there are simply portions of the footing that are assumed to be unavailable to carry the applied load.

\[
B' = B - 2e_B \\
L' = L - 2e_L
\]

Where  
- \( B' \) = the effective width of the footing after the effect of eccentric loading has been removed  
- \( B \) = the actual width of the footing  
- \( e_B \) = the distance from the center of the width that the load is applied  
- \( L' \) = the effective length of the footing after the effect of eccentric loading has been removed  
- \( L \) = the actual length of the footing  
- \( e_L \) = the distance from the center of the length that the load is applied

Eccentric loading can occur in the B and/or the L dimension. Footings with eccentric loadings in both directions are said to be experiencing double eccentricity.

The load must be applied within middle third of footing to avoid uplift on the edge of footing.

Modifications for Groundwater

The location of the ground water table can affect the unit weights (\( \gamma_1 \) and \( \gamma_2 \)) of the soils used in the bearing capacity calculation.

The following conditions could occur:  
1. The top of the ground water table is \( \geq B \) below the bottom of the footing.  
   a. Use the total unit weights of the soils in calculations.  
2. The footing is completely submerged (water table is at or above the ground surface).
a. Use the effective (aka buoyant) unit weights of the soils in calculations.

3. The top of the water table is at the bottom of the footing.
   a. Use the total unit weight for the soil above the base of the footing, \( \gamma_1 \) and the effective unit weight for the soil below the bottom of the footing, \( \gamma'_2 \).

4. The top of the water table is below the bottom of the footing, but higher than a depth of \( B \) below the bottom of the footing.
   a. Use the total unit weight of the soil above the bottom of the footing.
   b. Use a weighted average of the soil above and below the water table beneath the bottom of the footing. For example, if the water table starts at a depth of 0.75\( B \) beneath the foundation then the unit weight for \( \gamma_2 \) used in the equations would be 0.75\( \gamma_2 \) + 0.25\( \gamma'_2 \).

5. The water table is above the bottom of the footing, but does not extend to the ground surface above the footing.
   a. Use the weighted average method outlined in 4b above, but apply it to the soil above the bottom of the footing.
   b. Use \( \gamma'_2 \) for the soil beneath the footing.

As a rule of thumb, shallow foundations that are submerged will have about \( \frac{1}{2} \) of the capacity as those where the groundwater table is greater than or equal to a depth of \( B \) below the bottom of the footing.

---

Schematic for groundwater table.
Important Special Cases

$\phi = 0$ analysis for a footing on saturated clay:

The critical loading case for a footing built on saturated clay is rapid loading of the footing. Under rapid loading conditions the excess pore water pressures do not have a chance to dissipate. As such, a total stress, rather than an effective stress analysis is conducted. **In a total stress analysis always use total unit weights $\gamma$ rather than effective unit weights.**

Because $\phi = 0; N_c = 5.14, N_q = 1$ and $N_\gamma = 0$

The bearing capacity formula reduces to:

$$q_{ult} = c \ N_c \ s_c \ i_c \ d_c + \gamma_1 D_f \ s_q \ i_q \ d_q$$

Compute shape, inclination factors using $\phi = 0$ in formulas.

$$c = S_u$$

**Analysis in soils were $c = 0$**

There is no cohesion is soils such as uncemented sands. In this case uncemented does not refer to Portland cement, but rather naturally occurring cementation. Without cohesion, the bearing capacity equation reduces to:

$$q_{ult} = \gamma_1 D_f \ N_q \ s_q \ i_q \ d_q + (1/2) \ B \ \gamma_2 \ N_\gamma \ s_\gamma \ i_\gamma \ d_\gamma$$
Exercise 31.

For the strip footing on a saturated clay shown below, determine the maximum allowable column load $P_a$ to maintain a factor of safety of 3.

$$P_a \leq 1970 \text{ psf}$$

Note: Footing on sat. clay with load applied rapidly. No friction term.

Saturated Clay

$\gamma = 128 \text{ pcf}$

$S_u = 1,000 \text{ psf}$

Equations

$$q_{\text{net}} = \frac{q_{\text{ult}}(\text{net})}{FS}$$

$$q_{\text{ult}}(\text{net}) = q_{\text{ult}} - q$$

$\gamma = \gamma' + D_f$

$c = C N_c S_c i_c d_c + ...$

Friction term

$... + \gamma' D_f N_q S_q i_g d_g + ...$

Surchage term

$... + (1/2) B \gamma' d_g + ...$

$\gamma = \gamma' D_f N_q S_q i_g d_g + ...$

Solution

$$q_{\text{ult}} = (1000 \text{ psf}) (5.14) (1) (1) (1.15) + (128 - 62.4 \text{ pcf})(1.0)(1) (1) (1) + (1/2)(8 \text{ ft})(128 - 62.4 \text{ pcf})(0.5)(1) (1) (1) (1)$$

$$= 5911 \text{ psf} + 393.6 = 6304.6 \text{ psf}$$

$\gamma = (128 - 62.4)(1.0) = 393.6 \text{ psf}$

$\gamma_{\text{ult}}(\text{net}) = 6304.6 - 393.6 = 5911 \text{ psf}$

$$q_{\text{ult}}(\text{net}) = \frac{5911 \text{ psf}}{3}$$

$$q_{\text{a}(\text{net})} = 1970 \text{ psf}$$

Strip footing - all shape factors = 1
Vertical load - all inclined load factors = 1
Embedment - $d_f = 1$

$d_c = 1 + 0.2(1)(100 \text{ ft/80 ft}) = 1.15$
Exercise 32.

For the square footing on sand shown below, determine the maximum allowable column load $P_a$ to maintain a factor of safety of 3 for conditions of a deep groundwater table. You may neglect the depth factor effect; i.e. assume $d_q = d_r = 1$.

$$P_a = 15,579 \text{ psf}$$

No cohesion term

Equations

$$q_{ult(net)} = \frac{q_{ult(net)}}{FS}$$

$$q_{ult(net)} = q_{ult} - q$$

$$q = \gamma \cdot D_f$$

Since sand $\Rightarrow c = 0$

No cohesion term

So, $$q_{ult} = \gamma \cdot D_f \cdot N_q + \frac{1}{2} \beta \gamma^2 \cdot N_y$$

all shape and inclined load factors $= 1$

at $\phi = 35^\circ$, $N_q = 33.5$, $N_r = 37.8$

Solution

$$q_{ult} = (135 \text{ pcf}) \cdot (6 \text{ ft}) \cdot (33.5) + \frac{1}{2} (8 \text{ ft}) (135 \text{ pcf}) \cdot (37.8)$$

$$q_{ult} = 27135 + 120412 = 47547 \text{ psf}$$

$$q = (135 \text{ pcf}) (6 \text{ ft}) = 810 \text{ psf}$$

$$q_{ult(net)} = 47547 \text{ psf} - 810 \text{ psf}$$

$$q_{net} = 46737 \text{ psf}$$

$$\frac{q_{net}}{3} = \frac{15,579 \text{ psf}}{3}$$
Exercise 33.

For the square footing on sand shown below, determine the maximum allowable column load \( P_a \) to maintain a factor of safety of 3 for conditions of a groundwater table at the ground surface. You may neglect the depth factor effect; i.e. assume \( d_q = d_r = 1 \).

\[
P_a = 8378 \text{ psf}
\]

![Diagram of footing and groundwater table]

\( \gamma = 135 \text{ pcf} \)

\( \phi = 35 \text{ degrees} \)

\[ q_{ult} = \frac{q_{ult \text{ new}}}{FS} \]

\[ q_{ult \text{ new}} = q_{ult} - q_f \]

\[ q = \gamma \cdot D_c \]

\[ q_{ult} = \gamma \cdot D_c \cdot N_q + \frac{1}{2} B \cdot \gamma \cdot N_y \]

All shape and load inc. factors are 1.

\( N_q = 33.5 \)

\( N_y = 37.8 \)

\[ q_{ult} = (135 - 62.4)(64)(33.5) + \frac{1}{2}(435.6)(135 - 62.4)(37.8) \]

\[ q_{ult} = 14593 \text{ psf} + 10977 = 25570 \text{ psf} \]

\[ q = (135 - 62.4)(64) = 435.6 \text{ psf} \]

\[ q_{ult \text{ new}} = 25570 - 435.6 = 25134 \text{ psf} \]

\[ q_{a \text{ new}} = \frac{25134 \text{ psf}}{3} = 8378 \text{ psf} \]
Combined Rectangular Footing

Background
- Commonly needed for exterior columns, especially near property line
- A combined footing must be:
  (a) Large enough area such that $q_{\text{net}} < q_{\text{net(all)}}$
  (b) Proportioned for uniform pressure distribution

Design Procedure

Given data (see figure following page):
- $q_{\text{net(all)}}$ = net allowable bearing pressure
- $Q_1$ = exterior column load
- $Q_2$ = interior column load
- $L_2$ = distance from property line to center of Column 1
- $L_3$ = center to center spacing of Columns 1 and 2

Must find:
- $L$ = length of combined footing
- $B$ = width of combined footing

Step 1 – Determine required area of foundation, $A$:

$$A = \frac{(Q_1 + Q_2)}{q_{\text{net(all)}}}$$

Step 2 – Determine location of resultant of column loads, $X$:

$$X = \frac{Q_2 \cdot L_3}{(Q_1 + Q_2)}$$

Step 3 – Determine required footing length, $L$, for uniform pressure:

$$L = 2 \cdot (L_2 + X)$$

Step 4 – Determine distance, $L_1$:

$$L_1 = L - L_2 - L_3$$

Step 5 – Determine required footing width, $B$

$$B = \frac{A}{L}$$
Combined Rectangular Footing
Deep Foundations

Piles and drilled shafts are considered to be deep foundations. This means that the bottom of the pile or shaft is well below the ground surface compared to the diameter of the pile.

Deep foundations are used to extend through areas of weak soil to lower areas of stronger soil. The amount of settlement that piles or drilled shafts experience can be much less than strip or spread footings.

The analysis of pile capacity is broken into 2 distinct areas; piles that are founded in clay and piles that are founded in sand. This is not to say that there are not occasions in which a pile will have both sand and clay along its length. In those occasions, the portion of the pile in each layer will be analyzed separately.

Pile Capacity

The amount of load (capacity) that a pile can carry is made up of 2 components. The end bearing or tip resistance that is developed between the bottom end of the pile and the underlying soil and the skin friction that develops between the side of the pile and the adjacent soil.

Piles in clay

Pile Tip Resistance in Saturated Clays

\[ Q_p = N_c \times S_{up} \times A_p \]

Where  
\( Q_p \) = Tip resistance  
\( N_c \) = bearing capacity factor (typical) = 9  
\( S_{up} \) = undrained shear strength at level of pile tip  
\( A_p \) = cross-sectional area of pile
Pile Skin Resistance in Saturated Clays

There are several methods used to determine the skin resistance of piles in clay. The α-method is a common method.

α-method

\[ Q_s = \sum \alpha S_u p \Delta L \]

Where

- \( Q_s \) = skin resistance
- \( \alpha \) = adhesion factor (following pages)
- \( S_u \) = undrained shear strength
- \( p \) = perimeter (aka circumference) of pile
- \( \Delta L \) = length of vertical segment of pile

The α or adhesion factor is needed because there is a reduction in the amount of the undrained shear strength that is allowed to be used to support the side of the pile. In general, the lower the value of \( S_u \), the greater the percentage of the shear strength that is allowed to be used. This inverse relationship is due to the fact that softer clays are generally more cohesive (sticky) and will bind to the sides of the pile.

There is a summation sign in the above formula because the total skin resistance along the length of the pile is the sum of the skin resistances in each of the soil layers that the pile passes through.

Dealing with varying \( S_u \) within a soil profile

A soil layer can have a constant \( S_u \) within the layer. If this is the case then use the given value of \( S_u \) for skin friction calculations within the layer.

The undrained shear strength can vary within a layer. A common varying shear strength profile is one in which the shear strength linearly increases with depth within the layer. For example, the shear strength could be 100 at the top of the layer and increase linearly to 500 at the bottom of the layer. The average value of the shear strength within the layer (300 psf in this case) should be used in the skin resistance formula for this layer.
Typical $\alpha$ values

This chart shows how the alpha factor varies with unconfined compression stress in tons per square foot. Remember that unconfined compressive strength is twice the value of undrained shear strength.

$$q_u = 2Su = 2c$$
Exercise 34.

A 1-ft diameter pile is driven 50 ft into a normally consolidated clay. The soil strength varies linearly from zero at the ground surface to 1,000 psf at the pile tip. The estimated adhesion factor $\alpha=0.9$.

Estimated the total capacity of the pile.
Piles in sand

Pile Tip Resistance in Sands

The pile tip resistance in sands is greatly influenced by the vertical effective stress at the pile tip. The greater the vertical effective stress the greater the confining pressure around the tip and the harder it is for the pile to fail. There is a depth called the critical depth below which it is assumed that there is no longer a benefit to increasing vertical effective stress.

For pile tips that are above the critical depth:

\[ Q_p = A_p N_q q' < A_p q_L \]

Where
- \( Q_p \) = Tip resistance
- \( N_q \) = bearing capacity factor which is a function of \( \phi \), and can be found on the attached figure
- \( q' \) = in situ vertical effective stress at level of pile tip
- \( A_p \) = cross-sectional area of pile
- \( q_L \) = limiting tip pressure (below)

Limiting Tip Pressure:

\[ q_L \text{ (psf)} = 1000 N_q \tan \phi \]
Pile Skin Resistance in Sands

\[ Q_s = \Sigma p \Delta L f \]

Where
\[ Q_s = \text{skin resistance} \]
\[ p = \text{perimeter of pile} \]
\[ \Delta L = \text{length of vertical segment of pile} \]
\[ f = \text{unit frictional resistance} \]

As with the tip resistance, the skin resistance in sand is a function of the vertical effective stress. There is a critical depth below which it is assumed that the vertical effective stress remains constant. The critical depth is usually assumed to range from between 10 to 20 pile diameters. In other words, if a pile is 1 foot in diameter then the critical depth would range from 10 to 20 feet.

Unit friction resistance \((f)\)

\[ f = K \sigma_{v0}' \tan \delta < f_{\text{max}} \]

Where
\[ K = \text{earth pressure coefficient, which depends on method of pile installation. This is not the Rankine or Coulomb coefficient} \]
\[ \sigma_{v0}' = \text{vertical effective stress} \]
\[ \delta = \text{soil-pile interface friction angle} \]

The product of \(K\) and \(\sigma_{v0}'\) can be thought of as the horizontal force that is grabbing onto the side of the pile. The force is normal to the pile and is reduced by the coefficient of friction, \(\delta\) between the pile and the soil.

Maximum Unit Frictional Resistance:

\[ f_{\text{max}} = f \text{ at the critical depth} \]

Unless the problem statement specifically provides you with information dealing with the critical depth, do not limit the capacity of the pile.
Exercise 35.

A 1.5 ft diameter pile is driven 50 ft into the deposit shown below. Estimate the capacity of the pile assuming tip bearing resistance factor \( N_q = 120 \), and earth pressure coefficient \( K = 1.0 \) and a soil-pile interface friction angle \( \delta = 22 \) degrees. Neglect the side resistance from the clay in the calculation. Unit weights of the materials are as follows:

Clay: \( \gamma = 130 \) pcf
Sand: \( \gamma = 135 \) pcf

![Diagram of pile and deposit]
Pile Groups – Saturated Clay

Piles that are spaced closely together may tend to act as a group rather than individually. The capacity of the group may be more or less than the sum of the capacities of each individual pile. **The maximum capacity is the SMALLER of the group effect or the sum of the capacities of the individual piles.**

Definitions

The following definitions correspond to the diagram on the following page.

- \( D \) = pile diameter
- \( d \) = pile spacing (center to center)
- \( n_1 \) = # piles in x-direction
- \( n_2 \) = # piles in y-direction
- \( Q_{sp} \) = computed capacity of single pile
- \( Q_g \) = pile group capacity
- \( B_g \) = width of equivalent block
- \( L_g \) = length of equivalent block
- \( L \) = pile length

Procedure

1. Compute capacity of group assuming no interaction between piles:

\[
Q_g = Q_{sp} \ n_1 \ n_2
\]

\( Q_{sp} \) is calculated as shown earlier for single pile

2. Compute capacity assuming piles act as a block with dimensions \( L_g \) and \( B_g \) as shown in figure on following page.

\[
Q_g = L_g \ B_g \ S_u \ N_{c^*} + \Sigma \ 2 \ (L_g + B_g) \ S_u \ \Delta L
\]

Bearing factor \( N_{c^*} \) is shown in attached graph.

3. Take lower of values computed by 1 and 2 as group capacity.
Block has dimension $L_g$ into page.

Pile Group Acting as Equivalent Block
Bearing Capacity Factors for Pile Group Acting as Block.
Exercise 36.

A 3x3 group of 1-ft diameter piles is driven at 3-ft centers into the clay layer shown in the attached figure. Compute the capacity and efficiency of the pile group. Assume an adhesion factor $\alpha = 0.9$ in the single pile capacity calculations.
Site Investigations Overview

Sampling/Testing
Disturbed samples are ok for classification but can not be used for determining strength.

Examples of disturbed samples:
- Solid stem auger
- Bucket auger
- Split spoon sampler

Undisturbed samples are needed for lab testing of engineering properties such as permeability and strength.

Examples of undisturbed samples:
- Shelby tube (think sharpened muffler pipe)
- Fixed piston
- Hollow stem auger
- Pitcher, Osterberg, etc.

Sands are very difficult to sample without disturbance so we generally rely on in-situ tests

Calculation for disturbed versus undisturbed sample by Area Ratio (Ar)

One way to tell whether a particular sampling device collects disturbed or undisturbed samples is to compare the outside diameter to the inside diameter:

\[ A_r = \frac{D_o^2 - D_i^2}{D_i^2} \times 100\% \]

Where
- \( D_o \) = the outside diameter of the sampling tube
- \( D_i \) = the inside diameter of the sampling tube

If \( A_r \) is \( \leq \) then the sample can be considered undisturbed.
In-Situ (Measure properties in-place)

Can include penetration testing and geophysical testing

Drilling method also important: mud rotary usually preferred

**In Situ Testing**

**Standard Penetration Test (SPT)**
- Split spoon sampler (thick-walled)
- 140-lb hammer dropped 30 inches
- conventional cathead system – about 60% energy efficiency
- sampler driven in three 6-inch intervals
- sample extracted for visual/lab classification
- \( N_F \) = number of blows for last two 6-inch intervals
- \( N_F \) correlates to friction angle, modulus, relative density in sands
- \( N_F \) not very reliable for estimating clay properties (but sometimes used)
- Blow counts often corrected for stress level: \( N_1 = C_N N_F \)

**Correction factors** \( C_N \)
- Liao and Whitman \( C_N = (1/\sigma')^{1/2} \)
- Skempton \( C_N = 2/(1 + \sigma') \)
- Seed et al. \( C_N = 1-1.25 \log (\sigma'/1 \text{ tsf}) \)
- Peck et al. \( C_N = 0.77 \log (20 \text{ tsf/ } \sigma') \)

\( \sigma' \) is the vertical effective stress at the depth that the SPT test was conducted and is expressed in tons per square foot for all 4 correction factors.
Comparison of SPT N to Dr and Φ

The values in the following table are for uncorrected SPT N values rather than for corrected (N₁) values.

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>SPT N</th>
<th>Dr</th>
<th>Peck Φ</th>
<th>Meyerhof Φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very loose sand</td>
<td>&lt; 4</td>
<td>&lt; 0.2</td>
<td>&lt; 29</td>
<td>&lt; 30</td>
</tr>
<tr>
<td>Loose sand</td>
<td>4 to 10</td>
<td>0.2 to 0.4</td>
<td>29 to 30</td>
<td>30 to 35</td>
</tr>
<tr>
<td>Medium dense sand</td>
<td>10 to 30</td>
<td>0.4 to 0.6</td>
<td>30 to 36</td>
<td>35 to 40</td>
</tr>
<tr>
<td>Dense sand</td>
<td>30 to 50</td>
<td>0.6 to 0.8</td>
<td>36 to 41</td>
<td>40 to 45</td>
</tr>
<tr>
<td>Very dense sand</td>
<td>&gt; 50</td>
<td>&gt; 0.8</td>
<td>&gt; 41</td>
<td>&gt; 45</td>
</tr>
</tbody>
</table>

Cone Penetration Test (CPT)
- Used in sands or clays
- No soil sample
- 10 sq. cm probe advanced at 2 cm/s
- Measure tip resistance $q_c$ and sleeve friction $f_c$
- Undrained shear strength in clays:

$$S_u = \frac{(q_c - \sigma_0)}{N_K}$$

$\sigma_0 =$ total overburden stress
$N_K =$ bearing factor =10-20

- Sands – can correlate $q_c$ to friction angle, relative density or SPT-$N_F$

Rock Core Sampling

Samples of rock are removed from their parent material by drilling (coring) into the rock with a hollow coring bit. There are 2 ratios that provide an indication of the quality of the sample.
Recovery Ratio ($L_r$)

The Recovery Ratio, $L_r$ provides an indication of how much of the rock was collected for a given core attempt.

$L_r = \ \frac{\text{actual length of recovered sample}}{\text{theoretical length of the recovered sample}}$

Where: The actual length of the sample is found by measuring the pieces of the core that were brought to the surface.

The theoretical length of the recovered sample is the intended length of the core based on the distance of advance of the drill.

A low recovery ratio could be due to fractures in the rock that allow the sample to break apart and fall out of the core bit.

Rock Quality Designation (RQD)

The RQD provides a measure for the quality of the rock sample. Highly fractured rock is generally not a good foundation material due to the potential for crushing by the applied load. Poor quality rock will also have a high permeability. It is theoretically possible to pressure grout fractured rock, but the success of the program will depend upon the interconnectivity of the fractures.

$$\text{RQD} = \sum_{\text{length of pieces of core } \geq 10\text{cm}} \frac{\text{length of core advance}}{\text{length of core advance}}$$

The pieces of the core used in the RQD calculation must have intact (unbroken) lengths of at least 10 cm (about 4 inches). The length of each piece is measured along the centerline.

The length of the core advance is the same as in the recovery ratio.
<table>
<thead>
<tr>
<th>RQD</th>
<th>Rock Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.25</td>
<td>Very Poor</td>
</tr>
<tr>
<td>0.25 to 0.5</td>
<td>Poor</td>
</tr>
<tr>
<td>0.5 to 0.75</td>
<td>Fair</td>
</tr>
<tr>
<td>0.75 to 0.9</td>
<td>Good</td>
</tr>
<tr>
<td>0.9 to 1.0</td>
<td>Excellent</td>
</tr>
</tbody>
</table>
Exercise 37.

A standard penetration test is conducted at a depth of 30 ft in a sand having a unit weight $\gamma = 135$ pcf. The groundwater level is at a depth of 10 ft. The following blow counts are recorded: 12, 16, 18.

Compute the corrected standard penetration resistance $N_1$, using the correction by Peck et al.

Exercise 38.

A cone penetration test records a tip resistance $q_c = 12,950$ psf at a depth of 25 feet in a clay having a total unit weight $\gamma = 128$ pcf. Compute the undrained shear strength of the clay assuming a cone tip factor $N_k = 15$. 
Seismic Fundamentals

- Two major seismic issues in relation to geotechnical engineering:

  Site response: soil deposits can amplify or attenuate earthquake motions
  Liquefaction: dramatic loss of soil strength due to cyclic loading

- Inputs seismologists/geologists supply to geotechnical engineer:
  Design earthquake Magnitude, M
  Epicentral distance
  From attenuation relations, peak acceleration $a_{\text{max}}$

- Soils susceptible to liquefaction:
  Cohesionless coarse-grained – worst SP, SM; gravels can also liquefy
  Lower fines content (% passing # 200) => higher susceptibility
  Non-plastic, low plasticity - ML
  Loose / low relative density, $D_r < \sim 75\%$

- Factors affecting liquefaction potential (for simplified analysis)

  Penetration resistance, SPT-N value (measure of relative density)
  Magnitude of cyclic loading
Soil Liquefaction

Simplified liquefaction analysis

- Select earthquake magnitude and peak acceleration – M, $a_{\text{max}}$
- Compute corrected penetration resistance, $N_1 = C_N N$
- Obtain the cyclic resistance ratio (the cyclic stress required to cause liquefaction) for a magnitude 7.5 earthquake, $\text{CRR}_{7.5}$, from figure on following page
- Compute cyclic stress ratio, CSR

$$\text{CSR} = \frac{\tau_h}{\sigma'_0} = 0.65 \left( \frac{a_{\text{max}}}{g} \right) \left( \frac{\sigma_0}{\sigma'_0} \right) r_d$$

$(\tau_h/\sigma'_0) =$ cyclic stress ratio
$a_{\text{max}} =$ peak earthquake acceleration
$g =$ gravitational acceleration
$\sigma_0 =$ total vertical stress
$\sigma'_0 =$ effective vertical stress
$r_d =$ attenuation factor as function of depth (attached plot)

- Select magnitude scaling factor (MSF) for earthquakes other than M7.5:

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>MSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>2.20</td>
</tr>
<tr>
<td>6.0</td>
<td>1.76</td>
</tr>
<tr>
<td>6.5</td>
<td>1.44</td>
</tr>
<tr>
<td>7.0</td>
<td>1.19</td>
</tr>
<tr>
<td>7.5</td>
<td>1.00</td>
</tr>
<tr>
<td>8.0</td>
<td>0.84</td>
</tr>
<tr>
<td>8.5</td>
<td>0.72</td>
</tr>
</tbody>
</table>

- Factor of safety against liquefaction:

$$\text{FS} = \frac{\text{CRR}_{7.5}}{\text{CSR}} \times \text{MSF}$$
Chart for evaluating liquefaction potential of sands for different earthquakes. From Seed and Idriss (1971).
Residual Strength

- After liquefaction soil strength is not zero.

- Residual strength after liquefaction can be estimated from blow counts as shown in plot below.

Relationship Between Corrected “Clean Sand” Blowcount and Undrained Residual Strength (Seed and Harder, 1991)
Exercise 39.

The design earthquake for a project with the soil profile shown below is a Magnitude 7.5 earthquake with a peak ground acceleration $a_{\text{max}} = 0.3g$. At a depth of 20 ft, the soil has an uncorrected SPT-N of 10 blows/ft.

a. Determine whether liquefaction will occur at a depth of 20 ft.
b. Assuming liquefaction occurs, estimate the residual strength of the soil at a depth of 20 ft.

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**Soil Profile Diagram**

- 20 ft depth
- 10 ft groundwater
- 10 ft groundwater
- 20 ft depth
- Sand $\gamma = 132$ pcf

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