Vertical Curves
Vertical Curves

• In vertical planes, to provide smooth transitions between grade lines of tangent sections.
• Almost always parabolic to provide constant rate of change of grade.
• Crest and sag curves.
Design Criteria

• Minimize cut and fill.
• Balance cut and fill.
• Maintain adequate drainage.
• Not to exceed max. Grade.
• Meet fixed elevations, other roads or bridges.
• Provide sufficient sight distance.
General Equation of a Vertical Parabolic Curve

• For a second order parabola:

\[ Y_p = a + bX_p + cX^2_p \]

• What is the physical meaning of: a, b, c?
Figure 25–2  Terms for a parabola.
Define:

- BVC = VPC       V = VPI       EVC = VPT
- Percent grades g1, g2, \( r = \) rate of change of grade = \( \frac{g_2 - g_1}{L} \)
  \( L \) in stations, \( g \) not divided by 100
- The curve length \( L \) in stations, is it horizontal or curved length?
- What is an equal tangent vertical curve?
Figure 25–3  Vertical parabolic curve relationships.
Equation of an Equal Tangent Vertical Parabolic Curve in Surveying Terminology

- \[ Y = Y_{BVC} + g_1 X + \left(\frac{r}{2}\right) X^2 \]  
  \( r \) is -ve for crest

- Note that the value \( \left\{ \frac{r}{2} X^2 \right\} \) is the offset from the tangent, the equation is called *tangent offset equation*
Vertical Curve Computation Using the Tangent Offset Equation

- Select the grades, and hence find V’s
- The designer defines L, sight distance maybe?
- Compute the station of BVC, from the station of V and L, then compute the station of EVC, add L/2 to V?
- Note that you are not trying to locate the curve in the horizontal plane, just compute the elevations
- The problem:
  - Given: g1, g2, station and elevation of V, and L
  - Required: Elevation at certain distances (stations)
Figure 25-1  Grade line and ground profile of a proposed highway section.
Figure 25-4  Crest curve of Example 25-1.
Slope = \( gI = 2\% \)
What is that mean?

\[
0.02 \times 270 = 5.4 \text{ ft}
\]
or 
\[
2 \times 2.70 = 5.4 \text{ ft}
\]
$g_1 = +3.00\%, g_2 = -2.40\%, V \text{ station is 46+70 and } V \text{ elevation is 853.48, } L = 600 \text{ ft, compute the curve for stakeout at full stations.}$

**Answer:**

$r = \frac{-2.40 - 3.00}{6} = -0.90\%$

BVC station =

EVC station =

Elevation of BVC = $853.48 - (3.00)(3) = 844.48$

For each point, compute $X$ and substitute in the equation below to compute $Y$: $Y = 844.48 + 3.00(X) + (-0.90/2)X^2$

For example, at station 44+00: $X = 0.3$,

Then, $X = 1.3, 2.3, 3.3, 4.3, 5.3$, end at station 49+70: $X = \ldots
High or Low Points on a Curve

- Why: sight distance, clearance, cover pipes, and investigate drainage.
- At the highest or lowest point, the tangent is horizontal, the derivative of $Y$ w.r.t $x = 0$.
- Deriving the general formula gives:
  - $X = \frac{g_1}{(g_1 - g_2)} = -\frac{g_1}{r}$ where: $X$ is the distance in stations from BVC to the high or low point.
- Substitute in the tangent offset equation to get the elevation of that point.
- Example 25-4: compute the station and elevation of the highest point on the curve in example 25-1
- Answer: $X = -3.00/-0.9 = 3.3333$ stations

Plug $X$ back into the equation get elevation $= 849.48$
Designing a Curve to Pass Through a Fixed Point

- **Given:** $g_1$, $g_2$, VPI station and elevation, a point $(P)$ elevation and station on the curve.
- **Required:** You need five values to design a curve: $g_1$, $g_2$, VPI station and elevation, and curve length. The only missing value is the length of the curve.
- **Solution:**
  - Substitute in the tangent offset formula, the only unknown is $L$:
    \[ Y = Y_{BVC} + g_1 X + \left(\frac{r}{2}\right) X^2 \]
Figure 25–8  Designing a parabolic curve to pass through a fixed point.
Solution:
Substitute in the tangent offset formula, the only unknown is $L$: $Y = Y_{BVC} + g_1 X + (r/2) X^2$

Remember that:

- $Y_{BVC} = Y_V - g_1 (L/2)$, only $L$ is unknown
- $X$ is distance from BVC in stations, it is not the given station of $P$, to compute it, add or subtract the distance $V-P$ to/from $L/2$.

$$X = (L/2) \pm (V-P)_{\text{stations}}$$

Since you are given stations of VPI and the unknown point, you should be able to tell whether to add or to subtract.

- $r = (g_2 - g_1)/ L$
Example 25-5:

..g1 = -4.00, g2 = +3.80 %, V station is 52+00 and elevation is 1261.5 ft, the curve passes by point P at station 53+50 and elevation 1271.2 ft.

Answer (sag curve)

\[ X = \frac{L}{2} + 1.5 \]

Then:

\[ 1271.2 = \left\{ 1261.5 + 4.00 \frac{L}{2} \right\} + \left\{ -4.00 \frac{L}{2} \right\} + \left\{ \frac{3.8 + 4.00}{2L} \frac{L}{2} + 1.5 \right\} \]

\[ 0.975 L^2 - 9.85 L + 8.775 = 0 \]

Then,

\[ L = 9.1152 \text{ stations} \]
Sight Distance

- SSD : is the sum of two distances: perception reaction time, and the vehicle stopping distance.
- The length of the curve should provide enough SSD at design speed, and minimize cut and fill if possible.
- AASHTO design standards (2004): $H_1 = 1.08 \text{m}$, driver’s eye height, and
- $H_2 = 0.6 \text{ m}$, roadway object height.
- SSD given in tables.

fig 3.6, 3-6 mannering
Safe Length of Crest Curve

- To determine the safe length of a curve:
  - compute the SSD, or use tables, according to design conditions.
  - Compute (A): the absolute difference in grade.
  - Apply one of the formulas, neglecting the sign of (A):
    - \( L_m = 2 \, \text{SSD} - \left\{ \frac{200(\sqrt{H_1} - \sqrt{H_2})^2}{A} \right\} \)
      Substituting for \( H_1 = 1.08 \text{m} \) and \( H_2 = 0.6 \text{m} \) get
      \[
      L_m = 2 \, \text{SSD} - \left( \frac{658}{A} \right) \quad \text{if SSD} > L
      \]
    - Similarly, \( L_m = A \left( \frac{\text{SSD}^2}{658} \right) \) if SSD < L
    - Assume that SSD < L first, then check the answer.
Example:
At one section of a highway an equal tangent vertical curve must be designed to connect grades of +1.0% and −2.0%. Determine the length of curve required assuming that the SSD = 220.6m.

Answer:
Assume that $L > SSD$ (general assumption), then:

$L_m = A \left( \frac{SSD^2}{658} \right) = 3(220.6)^2 / 658 = 221.9m$

Since 221.9 m > 220.6 m so the assumption that

$L > SSD$ is correct.
Two vertical curves can be set-out at the same time from either BVC 1 or BVC 2. The direction of each of the Centerlines curves is shown to the left. Assume that the BM for elevation is BVC and IS NOT 20

Last tree