P.E. Civil Exam Review: Geometric Design

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Horizontal Curves
Back tangent

Forward tangent

T

T
P.C. or T.C.

Back tangent

Forward tangent

T

T

P.C. or T.C.

P.T.
Back tangent

Forward tangent

Back tangent

Forward tangent

P.I.

I = intersection angle

E

M

L = curve length

M

R

R

P.I.

P.C. or T.C.

P.T.

LC = long chord
Horizontal Curves:

EQUATIONS
Degree of curvature (D), chord basis:

\[
R = \frac{50}{\sin\left(\frac{1}{2}D\right)}
\]
Curve Equations: arc basis

\[ R = \frac{5729.58}{D} \]
Degree of curvature (D), arc basis:

\[ 2\pi R = \left( \frac{360^\circ}{D} \right) (100) \quad \rightarrow \quad R = \left( \frac{360^\circ}{D} \right) \left( \frac{100}{2\pi} \right) \quad \rightarrow \quad R = \frac{5729.58}{D} \]
Curve Equations:

\[ T = R \tan \left( \frac{1}{2} I \right) \]
Curve Equations:

\[ LC = 2R \sin \left( \frac{1}{2} I \right) \]
P. I. or P.C. or T.C.

R = long chord

L = curve length

E = intersection angle

I = intersection angle

T = tangent

M = middle point

P.C. or T.C.

P.T.

Back tangent

Forward tangent

Curve Equations:

\[ E = (T) \tan \left( \frac{I}{4} \right) \]

\[ E = R \left[ \sec \left( \frac{I}{2} \right) - 1 \right] \]

\[ E = R \left[ \frac{1}{\cos \frac{I}{2}} - 1 \right] \]
Curve Equations:

\[ M = R \left[ 1 - \cos \left( \frac{I}{2} \right) \right] \]
Curve Equations:

\[ L = \frac{100I}{D} \]
Horizontal Curves Equations

\[ R = \frac{5729.58}{D} \]

\[ R = \frac{50}{\sin\left(\frac{1}{2}D\right)} \]

\[ M = R \left[ 1 - \cos\left(\frac{I}{2}\right) \right] \]

\[ T = R \tan\left(\frac{1}{2}I\right) \]

\[ L = \frac{100I}{D} \]

\[ LC = 2R \sin\left(\frac{1}{2}I\right) \]

\[ E = (T) \tan\left(\frac{I}{4}\right) \]

\[ E = R \left[ \frac{1}{\cos\left(\frac{I}{2}\right)} - 1 \right] \]
Degree of curvature, chord basis. The degree of curvature is defined as the central angle subtended by a chord of 100 ft.

\[ R = \frac{50}{\sin\left(\frac{1}{2} D\right)} \]
Degree of curvature, arc basis. The degree of curvature is defined as the central angle of a circle which will subtend an arc of 100 ft.

\[ R = \frac{360}{D} \left( \frac{100}{2\pi} \right) = \frac{5729.8}{D} \]
Example of a sharp curve vs. a flat curve.

A sharp curve has a small $D$ and a large $R$.

A flat curve has a large $D$ and a small $R$. 
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.
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(a) What is the length of the curve?

(b) What is the station of the PC?

(c) What is the station of the PT?

(d) What is the interior angle at the PI?

(e) What is the tangent distance from the PI to the PC?

(f) What is the long chord distance?

(g) What is the external distance?

(h) What is the degree of the curve (arc basis)?

(i) What is the degree of the curve (chord basis)?
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

\[ R = \frac{5729.58}{D} \quad \rightarrow \]

\[ D = \frac{5729.58}{800} \quad \rightarrow \]

\[ D = 7.162° \]
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(a) What is the length of the curve?

\[ L = \frac{100I}{D} \rightarrow L = \frac{100(95°)}{7.162} \rightarrow L = 1326.44 \text{ ft} \]

(b) What is the station of the PC?

\[ T = R \tan\left(\frac{I}{2}\right) \]

\[ T = 800 \tan(47.5°) \rightarrow T = 873.05 \text{ ft} \]

\[ PC = (37 + 00) - (8 + 73.05) \rightarrow PC = 28 + 27 \text{ station} \]
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(c) What is the station of the PT?

\[ PC = 28 + 27 \text{ \, station} \]
\[ L = 1326.44 \text{ \, ft} \]
\[ PT = (28 + 27) + (13 + 26.44) \rightarrow PT = 41 + 53.44 \text{ \, station} \]
EXAMPLE PROBLEM:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(d) What is the interior angle at the PI?

\[ 180° - 95° = 85° \]
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(e) What is the tangent distance from the PI to the PC?

\[ T = R \tan\left(\frac{I}{2}\right) \quad \rightarrow \quad T = 873.05 \text{ ft} \]

(f) What is the long chord distance?

\[ LC = 2R \sin\left(\frac{I}{2}\right) = 2(800) \sin\left(\frac{95°}{2}\right) \quad \rightarrow \quad LC = 1179 \text{ ft} \]
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(g) What is the external distance?

\[ E = R\left[\sec\left(\frac{I}{2}\right) - 1\right] = 800\left[\sec\left(\frac{95}{2}\right) - 1\right] \rightarrow E = 384 \text{ ft} \]

\[ E = (T) \tan\left(\frac{I}{4}\right) = 384 \text{ ft} \]

\[ E = R\left[\frac{1}{\cos\frac{I}{2}} - 1\right] = 384 \text{ ft} \]
EXAMPLE PROBLEM 1:

The bearings of two tangents connected by a horizontal circular curve are N50°E and S35°E, respectively. The tangents intersect at station 37+00. The curve radius is 800 ft.

(h) What is the degree of the curve (arc basis)?

\[
D = \frac{5729.58}{R} = \frac{5729.58}{800} \rightarrow D = 7.16°
\]

(i) What is the degree of the curve (chord basis)?

\[
R = \frac{50}{\sin\left(\frac{1}{2} D\right)} \quad \sin\left(\frac{1}{2} D\right) = \frac{50}{800} \quad D = 2\sin^{-1}(0.06)
\]

\[
D = 7.16°
\]
Example Problem 2

Convert the following angle to degrees

\[39^{\text{deg}} \ 41^{\text{min}} \ 54^{\text{sec}}\]

Solving:
\[54/60 = 0.9 \ \text{min}\]
\[41.9/60=0.69 \ \text{deg}\]

Answer = 39.69 degrees
Example Problem 3

Express the following angle in terms of degrees, minutes, and seconds

$91.74\,\text{degrees}$

Solving;

$0.74\, (60) = 44.4\, \text{minutes}$

$0.4\, (60) = 24.0\, \text{seconds}$

Answer: $91^{\text{deg}}\, 44^{\text{min}}\, 24^{\text{sec}}$
EXAMPLE PROBLEM 4:

The two tangents shown intersect 2000 ft beyond Station 10+00. The back tangent has a bearing of N 45°00’00” W and the forward tangent has a bearing of N15°00’00” E. The decision has been made to design a 3000 ft radius horizontal curve between the two tangents.

(a) What is the central angle of the curve?
(b) Compute the tangent distance and length of curve.
(c) What is the station of the PC?
(d) What is the station of the PT?
(e) What is the degree of curvature of this curve using the arc definition?
EXAMPLE PROBLEM 4:

1. What is the central angle for this curve?
(a) 30 degrees
(b) 65 degrees
(c) 45 degrees
(d) 60 degrees
(e) 15 degrees

2. What is the tangent distance for this curve?
   a) 1551.70 ft
   b) 1732.05 ft
   c) 1673.29 ft
   d) 2000.00 ft
   e) 3000.00 ft
EXAMPLE PROBLEM 2:

3) What is the length of this curve?
(a) 2000.00 ft
(b) 3190.60 ft
(c) 5729.60 ft
(d) 3000.00 ft
(e) 1047.20 ft

4) What is the station of the PC?
(a) 30 +00.00
(b) 20+00.00
(c) 17+32.05
(d) 19+32.05
(e) 12+67.95
EXAMPLE PROBLEM 4:

5) What is the station of the PT?
   (a) 47+32.05
   (b) 44+09.55
   (c) 43+32.05
   (d) 46+32.05
   (e) 42+09.55

6) What is the degree of curvature of this curve using the arc definition?
   a) 60 degrees
   b) 3.5 degrees
   c) 2.9 degrees
   d) 1.9 degrees
   e) 3.9 degrees
EXAMPLE PROBLEM 5:

The long chord of a circular curve is 600 feet, the intersection angle is 110°. Find the radius. The forward tangent of this curve needs to be moved in 5 feet due to a right of way dispute. What radius curve would you specify to hold the B.C. at the same location as in the original curve?
I = 110°

R = ?

T

T'

P.C.

P.T.

C

600'

110°

5'

110°

Stations →
EXAMPLE PROBLEM 5:

The long chord of a circular curve is 600 feet. The intersection angle is 110 degrees. What is the radius of this curve? The forward tangent of this curve needs to be moved in 5 feet due to a right of way dispute. What radius curve would you specify to hold the PC at the same location as in original first curve.

We know: \( \text{L.C.} = 600 \text{ ft} \)

\[
LC = 2R \sin \left( \frac{I}{2} \right)
\]

Then

\[
R = \frac{600}{2 \sin \left( \frac{110^0}{2} \right)} = 366.23 \text{ ft}
\]

\[
T = R \tan \left( \frac{I}{2} \right) = 366.23 \tan \left( \frac{110^0}{2} \right) = 523.03 \text{ ft}
\]
EXAMPLE PROBLEM 5:

\[ X = \frac{5}{\sin 70^0} \]

\[ X = 5.32 \]

\[ T_{new} = 523.03 - 5.32 = 517.71 \text{ ft} \]

\[ R_{new \text{ curve}} = \frac{T_{new}}{\tan 55^o} = \frac{517.71}{1.428} = 362.50 \text{ ft} \]

*Please note that the answer is NOT \( (366.23 - 5) \)*
EXAMPLE PROBLEM 5:

The alignment of a proposed highway was adjusted in order to avoid a critical obstacle. This was accomplished by moving the forward tangent 120 ft forward.

You are asked to find the radius of the new curve if it begins at the same location as the PC of the initial curve.

You need to also determine the stations for PC and PT for the new curve alignment.
I = 65°

Original Tangent

6° Curve

Initial Curve

P. I. Sta. 85+71.92

Tangent moved forward

I = 65°

120'

New P.T.

120’
I = 65°

6° Curve

Initial Curve

Original Tangent

P.I. Sta. 85+71.92

120'

l = 65°

Proposed new tangent

New P.T.

120'
EXAMPLE PROBLEM 5:

For the original curve, find $R$ and $T$:

$$R = \frac{5729.58}{D} = \frac{5729.58}{6} = 954.93 \text{ ft}$$

$$T = R \tan \frac{I}{2} = \left( 954.93 \right) \left( 0.637 \right) = 608.36 \text{ ft}$$

PC station for the original curve is:

$$(85 + 71.92) - (6 + 8.36) = 79 + 63.56$$
\[ X = \frac{120}{\sin 65^0} = 132.40 \text{ ft} \]
EXAMPLE PROBLEM 5:

\[ T_{\text{new}} = 608.36 + 132.40 = 740.76 \text{ ft} \]

\[ R_{\text{new curve}} = \frac{T_{\text{new}}}{\tan\left(\frac{65^0}{2}\right)} = \frac{740.76}{0.637} = 1162.76 \text{ ft} \]

\[ D_{\text{new}} = \frac{5729.58}{1162.76} = 4.93^0 \]

\[ L_{\text{new curve}} = \frac{100 \cdot I}{D_{\text{new}}} = \frac{100 \left(65^0\right)}{4.93^0} = 1319.10 \text{ ft} \]

PT Station = Station PC + L

New PT Station = (79 + 63.56) + (13 + 19.10) = 92 + 82.66
Vertical Curves
P.V.I.

P.V.C.

P.V.T.

Back tangent

Forward tangent

LC = long chord
Tangent offsets

Back tangent

Forward tangent

P.V.I.

P.V.C.

P.V.T.

B

C

LC = long chord

52
PROPERTIES OF A PARABOLA:
Applied to Vertical Curve Analysis
PROPERTIES OF A PARABOLA:

1. The curve elevation at its midpoint is halfway from the elevation at the P.V.I. to the elevation at the midpoint of the long chord.

\[
E = \frac{1}{2} \left( PVI \text{ elevation} - \frac{PVC \text{ elevation} + PVT \text{ elevation}}{2} \right)
\]

The curve lies midway between the point of intersection of the grade lines and the middle point of the chord joining the BVC (beginning of vertical curve) and the EVC (end of vertical curve).

2. The tangent offsets vary as the square of the distance from the point of tangency

\[
\frac{Y}{x^2} = \frac{E}{T^2}
\]
PROPERTIES OF A PARABOLA:

\[ E = \frac{(g_1 - g_2)L}{8} \]

\[ r = \frac{g_2 - g_1}{L} = \text{rate of grade change} \]

\[ \frac{y}{x^2} = \frac{E}{T^2} \]
Example Problem 7

GIVEN:
- Station at PVI
- Elevation at PVI
- $g_1 = +4\%$
- $g_2 = -3\%$
- Curve Length 800 ft
Stations  
61  62  63  64  65  66  67  68  69

Elevations on grade lines

Stations

P.V.I.

264.20

P.V.C.

P.V.T.

B

C

g₁ = +4%

g₂ = -3%

7.00'
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![Diagram](image)

- $g_1 = +4\%$
- $g_2 = -3\%$
Calculations for example 7:

Elevation of PVI = 264.20 ft

Elevation of C = \( \frac{1}{2}(248.20+252.20) = 250.2 \) ft

Elevation of B = \( \frac{1}{2}(264.20+250.20) = 257.20 \) ft

\[
E = \frac{(g_1 - g_2)L}{8} = \frac{(4 - (-3))(8)}{8} = 7
\]

\[
r = \frac{(g_2 - g_1)}{L} = \frac{-3 - 4}{8} = -\frac{7}{8}
\]

rate of grade change
The tangent offsets vary as the square of the distance from the point of tangency.

\[ \frac{y}{x^2} = \frac{E}{T^2} \]
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\[
\frac{y}{x^2} = \frac{E}{T^2}
\]

\[
\frac{y}{3^2} = \frac{7}{4^2}
\]

\[
y = 3^2 \left[ \frac{7}{4^2} \right]
\]

\[
y = 3.94 \text{ tangent offset at station 64+00}
\]

\[
g_2 = -3\%
\]
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\[ y = 2^2 \left[ \frac{7}{4^2} \right] = 1.75 \]

\[ g_2 = -3\% \]
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Tangent offset at station 62+00

\[ y = 1^2 \left[ \frac{7}{4^2} \right] = 0.44 \]
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Diagram showing the stations and elevations on grade lines. The diagram includes the following annotations:

- $g_1 = +4\%$
- $g_2 = -3\%$
- Elevations at stations:
  - P.V.C.: 0.44'
  - B: 1.75'
  - P.V.I.: 3.94'
  - C: 7.00'

Points labeled:
- P.V.C.
- B
- C
- P.V.T.
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Diagram:
- P.V.C.: Initial point
- P.V.I.: Intermediate point
- P.V.T.: Terminal point
- B: Point on grade line
- C: Point on grade line

- $g_1 = +4\%$
- $g_2 = -3\%$

- Elevations at stations:
  - 248.20
  - 252.20
  - 256.20
  - 260.20
  - 264.20
  - 261.20
  - 258.20
  - 255.20
  - 252.20
### Elevations on grade lines

<table>
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<th>Elevations on curve</th>
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\[ g_1 = +4\% \]

\[ g_2 = -3\% \]
Highest and Lowest Points on Vertical Curves:

$X$ is the distance between PVC and station of high or Low point

$$X = \frac{-g_1}{r}$$

For the vertical curve in example problem 5:

$$r = \frac{g_2 - g_1}{L} = \frac{-3 - 4}{8} = -\frac{7}{8}$$

$$X = \frac{-g_1}{r} = \frac{-4}{-\frac{7}{8}} = +4.57 \text{ Stations from PVC}$$

Elevation at highest point Station 65+57 is equal to:

$$264.20 - (0.03)(57) - \frac{(3.43)^2}{(4)^2}(7.00) = 257.34 \text{ ft}$$
Calculations for the Elevation at highest point on the vertical curve which is at station 65+57:

Elevation of tangent line at station 65+57 is,
264.20-(.03)(57)= 264.20-1.7 = 262.49 ft

• Offset at station 65+57 is calculated by $\frac{7}{\left[4\right]} = y / \left[3.43\right]$

• Offset at station 65+57 is $y=5.147$ ft

• Elevation at station 65+57 on the vertical curve is $262.49 - 5.147$ which is equal to $257.34$ ft
Vertical Curve Problem #8
Station of BVC = 50 – 4 = station 46
Elevation of BVC = 550.97 + (4 x 6) = 574.97 ft
Station of EVC = 50 + 4 = station 54
Elevation of EVC = 550.97 + (4 x 2) = 558.97 ft
Vertical Curve Problem #8
Solutions

• Elevation of middle point of chord = \[ \frac{574.97 + 558.97}{2} = 566.97 \text{ ft} \]

• Offset to curve at intersection = \[ \frac{566.97 - 550.97}{2} = 8.00 \text{ ft} \]
Vertical Curve Problem #8
Solutions

• Offset at A and F = \left(\frac{1}{4}\right)^2 \times 8.00 = 0.50 \text{ ft}

• Offset at B and E = \left(\frac{2}{4}\right)^2 \times 8.00 = 2.00 \text{ ft}

• Offset at C and D = \left(\frac{3}{4}\right)^2 \times 8.00 = 4.50 \text{ ft}
Elevation of A = 574.97 − 6.00 = 568.97 ft
Elevation of B = 568.97 − 6.00 = 562.97 ft
Elevation of C = 562.97 − 6.00 = 556.97 ft
Elevation of V = 556.97 − 6.00 = 550.97 ft
Elevation of F = 558.97 − 2.00 = 556.97 ft
Elevation of E = 556.97 − 2.00 = 554.97 ft
Elevation of D = 554.97 − 2.00 = 552.97 ft
Elevation of V = 552.97 − 2.00 = 550.97 ft*
– *checks!
Vertical Curve Problem #8
Solutions

- Elevation of $a =$ station 47 $= 568.97 + 0.50 = 569.47$
- Elevation of $b =$ station 48 $= 562.97 + 2.00 = 564.97$
- Elevation of $c =$ station 49 $= 556.97 + 4.50 = 561.47$
- Elevation of $o =$ station 50 $= 550.97 + 8.00 = 558.97$
- Elevation of $d =$ station 51 $= 552.97 + 4.50 = 557.47$
- Elevation of $e =$ station 52 $= 554.97 + 2.00 = 556.97$
- Elevation of $f =$ station 53 $= 556.97 + 0.50 = 557.47$
### Vertical Curve Problem #8

#### Solutions

<table>
<thead>
<tr>
<th>STATION</th>
<th>TANGENT ELEVATION</th>
<th>OFFSET FROM TANGENT</th>
<th>ELEVATION CURVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BVC = 46</td>
<td>574.97</td>
<td>0</td>
<td>574.97</td>
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<tr>
<td>47</td>
<td>568.97</td>
<td>+0.50</td>
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<td>48</td>
<td>562.97</td>
<td>+2.00</td>
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<tr>
<td>49</td>
<td>556.97</td>
<td>+4.50</td>
<td>561.47</td>
</tr>
<tr>
<td>50</td>
<td>550.97</td>
<td>+8.00</td>
<td>558.97</td>
</tr>
<tr>
<td>51</td>
<td>552.97</td>
<td>+4.50</td>
<td>557.47</td>
</tr>
<tr>
<td>52</td>
<td>554.97</td>
<td>+2.00</td>
<td>556.97</td>
</tr>
<tr>
<td>53</td>
<td>556.97</td>
<td>+0.50</td>
<td>557.47</td>
</tr>
<tr>
<td>EVC = 54</td>
<td>558.97</td>
<td>0</td>
<td>558.97</td>
</tr>
</tbody>
</table>
SIGHT DISTANCE:

Minimum sight distance (s), where sight distance is less than the curve length (L) in stations and the height of driver’s eye (h) is 3.5 ft to 3.75 ft above pavement.

\[ s^2 = \frac{8Lh}{g_1 - g_2} \]

If the calculated sight distance (s) is greater than the curve length (L) then use the following equation:

\[ s = \frac{L}{2} + \frac{4h}{g_1 - g_2} \]
EXAMPLE PROBLEM 9:

Sight distance calculations for the vertical curve in problem 7:

\[ L = 800 \text{ ft} \quad g_1 = 4\% \quad g_2 = -3\% \quad h = 3.75 \text{ ft} \]

\[ s = \sqrt{\frac{8(8.00)(3.75)}{4 - (-3)}} = 5.8554 \text{ stations} \quad \rightarrow \quad s = 585.54 \text{ ft} \]

If the calculated sight distance (s) is greater than the curve length (L) then use the following equation:

\[ s = \frac{L}{2} + \frac{4h}{g_1 - g_2} \]
EXAMPLE PROBLEM 10:

A proposed 2-lane highway has a vertical alignment that is +3% grade intersecting a −2% grade at station 26+00 at an elevation of 228.00. The proposed alignment must bridge over an existing railroad track which crosses the proposed alignment at station 28+50. The elevation of the railroad track at the point of intersection is 195.00. The proposed highway alignment must have a vertical elevation difference of 26.0 ft at station 28+50 in order to satisfy vertical clearance requirements.

You are to determine the longest length of vertical curve, rounded to the nearest 100 ft, that will fulfill these criteria. Determine the station of the high point on the vertical curve.
g_1 = +3\% \\
g_2 = -2\% \\
26+00 \\
El. 228.00 \\
28+50 \\
El. 223 \\
El. 221 \\
clearance requirement = 26.0 ft. \\
El. 195.00
EXAMPLE PROBLEM 10:

For vertical curve to go through a fixed point, use the following formula:

\[
L = 2 \left( A + \frac{2(\text{offset})}{g_1 - g_2} \right) + 4 \sqrt{\frac{A(\text{offset})}{g_1 - g_2} + \left( \frac{\text{offset}}{g_1 - g_2} \right)^2}
\]

\[
\text{offset} = 2 \quad A = 2.5 \quad g_1 - g_2 = 5
\]

\[
L = 10.91 \text{ stations} \quad \text{therefore use } L = 1000 \text{ ft to maintain 26 ft minimum clearance}
\]

Station of high point

\[
x = -\frac{g_1}{r} = -\frac{-3}{-2 - 3} = 6 \text{ Stations from PVC}
\]
EXAMPLE PROBLEM 10:

Using design curve length $L=1000$ ft

Location of high point

$$x = -\frac{g_1}{r} = -\frac{-3}{-2 - 3} = 6 \quad Stations \ from \ PVC$$

Station at PVC = (26+00) – (5+00) = 21+00

Station of high point = (21+00) + (6+00) = 27+00
EXAMPLE PROBLEM 11:

The proposed highway is to cross another highway at right angles. The elevation of the proposed crossing has been established and a minimum vertical clearance of 25 ft. will be required between the proposed highway and the existing highway.

A. Determine the location and elevation of the low point on the existing vertical curve.

B. Determine the minimum station at which the crossing may be located.

C. Determine the maximum station at which the crossing may be located.
$g_1 = -4\%$

$g_2 = +3\%$

$\text{Existing Vert. Curve of other highway}$

$\text{Proposed Highway}$

El. 777

25’

El. 736

V.P.I.

82+00

88+00

94+00
EXAMPLE PROBLEM 11:

Calculate Low point on existing vertical curve

\[ x = - \frac{g_1}{r} = - \frac{-4}{3 - (-4)} = 6.8571 \text{ Stations from PVC} \]

Station at low point = (82+00) + (6+85.71) = 88+85.71

Elevation of low point is calculated to be 746.28 ft

Proposed highway has an elevation of 777 ft and required clearance is 25 ft, therefore, there exists two locations on the curve with elevation of 777 − 25 = 752 ft

By trial and error, we need to find minimum and maximum station where curve elevation is 752 ft
Existing Vert. Curve

Proposed Highway

El. 777

25'

El. 736

25'

V.P.I.

El. 736

V.P.I. 88+00

E.V.C. 94+00

P.V.C. 82+00

$g_1 = -4\%$

$g_2 = +3\%$

Low point
Trial and error for maximum station:

<table>
<thead>
<tr>
<th>Station</th>
<th>Tangent elevation</th>
<th>offset</th>
<th>Curve elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>93+00</td>
<td>751.00</td>
<td>0.29</td>
<td>751.29</td>
</tr>
<tr>
<td>94+00</td>
<td>754.00</td>
<td>0.00</td>
<td>754.00</td>
</tr>
<tr>
<td>93+30</td>
<td>751.90</td>
<td>0.14</td>
<td>752.04</td>
</tr>
<tr>
<td>93+29</td>
<td>751.87</td>
<td>0.15</td>
<td>752.02</td>
</tr>
<tr>
<td>93+28</td>
<td>751.84</td>
<td>0.15</td>
<td>751.99</td>
</tr>
<tr>
<td>93+28.5</td>
<td>751.855</td>
<td>0.15</td>
<td>752.00</td>
</tr>
</tbody>
</table>

Maximum Station turns out to be 93+28.5
Trial and error for minimum station:

<table>
<thead>
<tr>
<th>Station</th>
<th>Tangent elevation</th>
<th>offset</th>
<th>Curve elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>84+00</td>
<td>752.00</td>
<td>1.17</td>
<td>753.17</td>
</tr>
<tr>
<td>85+00</td>
<td>748.00</td>
<td>2.62</td>
<td>750.62</td>
</tr>
<tr>
<td>84+50</td>
<td>750.00</td>
<td>1.82</td>
<td>751.82</td>
</tr>
<tr>
<td>84+40</td>
<td>750.40</td>
<td>1.68</td>
<td>752.08</td>
</tr>
<tr>
<td>84+42</td>
<td>750.32</td>
<td>1.71</td>
<td>752.03</td>
</tr>
<tr>
<td>84+43</td>
<td>750.28</td>
<td>1.72</td>
<td>752.00</td>
</tr>
</tbody>
</table>

Minimum Station turns out to be 84+43
References:


Thank You.

Any Questions?

Good Luck!