Excavation and embankment (cut and fill)

**Excavation** = the removal of soil or rock from its natural location.

**Embankment** = the placement and compaction of layers of earth or rock to form a roadbed of the planned shape, density, and profile grade.

Various sections of a roadway design will require bringing in earth. Other sections will require earth to be removed. Earth that is brought in is considered **Fill** while earth that is removed is considered **Cut**. Generally, designers generate drawings called Cut and Fill Diagrams, which illustrate the cut or fill present at any given site. This drawing is quite standard, being no more than a graph with site location on the X-axis and fill being the positive range of the Y-axis while cut is the negative range of the Y-axis.

For the PE exam - cut-fill problems are really simple. There are two types of problems that could be asked.

1. What is the volume of the cut/fill area for a road between two stations?
2. What is the volume of the cut/fill area for a road between multiple stations?

In real life, mostly these problems are solved using computer programs. However, you don’t have the luxury doing the PE Exam so you need to know a few things,

The only things you need to know are

1. How to find the Area of the cross section
   - Usually given
   - Use Geometry
2. How to find the volume using the different techniques

   **Average end area Method**
   \[
   V = \frac{A_1 + A_2}{2} \cdot L
   \]

   **Prismoidal Method** – This is a more accurate formula, which takes out most of the error accrued by the average end area method.
   \[
   V_p = \frac{L(A_1 + 4A_m + A_2)}{6}
   \]

   **Pyramid Method** - If one end area has a value of zero, the earthwork volume can be considered a pyramid and the correct formula would be:
   \[
   V = \frac{AL}{3}
   \]
To figure out what is the total net fill between station 1+00 and 2+00.

Step 1: figure out total fill area per station. The area is usually given.
- STA 1+00 = Fill 1 area = 100ft$^2$
- STA 2+00 = Fill 2 area = 40ft$^2$

Step 2: Use formula to calculate the fill volume between stations
- Volume fill = (100ft$^2$+40ft$^2$)/2 x 100ft = 7000 ft$^3$

Step 4: Convert Cubic Feet to Cubic Yards
- CY soil = CF soil/27 = 7000 ft$^3$ /27 = 260 CY Fill

To figure out what is the total net cut/fill between station 1+00 and 2+00.

Step 1: figure out total cut/fill area per station. This is usually given.
- STA 1+00 = Cut 1 area = 85ft$^2$, Fill 1 area = 100ft$^2$
- STA 2+00 = Cut 2 area = 140ft$^2$, Fill 2 area = 20ft$^2$

Step 2: Use above formula to calculate the cut/fill volume between stations
- Volume cut = (85ft$^2$+140ft$^2$)/2 x 100ft = 11250 ft$^3$
- Volume fill = (100ft$^2$+20ft$^2$)/2 x 100ft = 6000 ft$^3$

Step 3: Find total net cut or fill between stations
- Total cut or fill = Volume cut – Volume fill
- Total cut = 11250ft$^3$ - 6000ft$^3$ = 5250 ft$^3$ Cut

Step 4: Convert Cubic Feet to Cubic Yards
- CY soil = CF soil/27 = 5250 ft$^3$ /27 = 194.44CY Cut
Cut and Fill between multiple Stations

<table>
<thead>
<tr>
<th>Station #</th>
<th>Cut Area</th>
<th>Fill Area</th>
<th>Cut Vol</th>
<th>Fill Vol</th>
<th>Net Vol</th>
<th>Cum Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>0+00</td>
<td>175</td>
<td>125</td>
<td>7300</td>
<td>6200</td>
<td>1100</td>
<td>1100</td>
</tr>
<tr>
<td>0+50</td>
<td>117</td>
<td>123</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1+00</td>
<td>238</td>
<td>250</td>
<td>8875</td>
<td>9325</td>
<td>-450</td>
<td>650</td>
</tr>
<tr>
<td>1+50</td>
<td>211</td>
<td>240</td>
<td>11225</td>
<td>12250</td>
<td>-1025</td>
<td>-375</td>
</tr>
<tr>
<td>2+00</td>
<td>198</td>
<td>180</td>
<td>10225</td>
<td>10500</td>
<td>-275</td>
<td>-650</td>
</tr>
<tr>
<td>2+50</td>
<td>140</td>
<td>141</td>
<td>8450</td>
<td>8025</td>
<td>425</td>
<td>-225</td>
</tr>
<tr>
<td>3+00</td>
<td>258</td>
<td>200</td>
<td>9950</td>
<td>8525</td>
<td>1425</td>
<td>1200</td>
</tr>
</tbody>
</table>

To figure out what is the total net cut/fill between stations 1+00 and 3+00.
Columns Station #, Cut Area, and Fill area will be given.

Step 1: Complete “Cut Vol”
Step 2: Complete the “Fill Vol”
Step 3: Complete the “Net Cut”
Step 4: Complete the Cum Cut
In constructing a new road, the design estimates the following areas for earthwork by the stations provided.

<table>
<thead>
<tr>
<th>Station</th>
<th>Area (ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>519+00</td>
<td>0</td>
</tr>
<tr>
<td>520+00</td>
<td>120</td>
</tr>
<tr>
<td>521+00</td>
<td>224</td>
</tr>
<tr>
<td>522+00</td>
<td>325</td>
</tr>
<tr>
<td>523+00</td>
<td>450</td>
</tr>
<tr>
<td>524+00</td>
<td>362</td>
</tr>
<tr>
<td>525+00</td>
<td>0</td>
</tr>
</tbody>
</table>

The cost for extracting the soil is $20 per yd³ moving the soil is $5 per yd³. The total cost to extract and move the estimated volume of earth, in thousands of dollars, is most nearly:

(A) 126
(B) 130
(C) 137
(D) 145

Solution

The end section areas are zero. Apply the pyramid method for the end segments, because their area is zero, and apply the end area method for the middle sections. The end area method incorporates the area between each pair of full stations. This assumes that the average represents the section as a whole and therefore the lengths between the end areas should be appropriate. Apply:

where:

- is volume in yd³
- is end area in ft²
- is end area in ft²
- is the length between \( A_1 \) and \( A_2 \) in feet
- is the conversion factor to transform \( ft² \) into \( yd³ \)
Be sure to check the lengths and do not assume they are all 100 feet although that is the common stationing for doing the end area method.

For the two ends, the pyramid method is applied:

where:

- $V$ is volume in yd$^3$
- $A$ is the non-zero end area in ft$^2$
- $L$ is length between $A_1$ and $A_2$ in feet

___ is the conversion factor to transform ft$^2$ into yd$^2$ and the pyramid constant

<table>
<thead>
<tr>
<th>Station</th>
<th>Area (ft$^2$)</th>
<th>Length (ft)</th>
<th>Volume (yd$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>519+00</td>
<td>0</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>520+00</td>
<td>120</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>521+00</td>
<td>224</td>
<td>100</td>
<td>$\frac{100 \times (224 + 325)}{54} = 1,017 \text{ yd}^3$</td>
</tr>
<tr>
<td>522+00</td>
<td>325</td>
<td>100</td>
<td>$\frac{100 \times (325 + 450)}{54} = 1,435 \text{ yd}^3$</td>
</tr>
<tr>
<td>523+00</td>
<td>450</td>
<td>100</td>
<td>$\frac{100 \times (450 + 362)}{54} = 1,504 \text{ yd}^3$</td>
</tr>
<tr>
<td>524+00</td>
<td>362</td>
<td>100</td>
<td>$\frac{100 \times 362}{81} = 447 \text{ yd}^3$</td>
</tr>
<tr>
<td>525+00</td>
<td>0</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Total Volume = $1,017 + 1,435 + 1,504 + 447 = \text{ yd}^3$

At $\$20$ for excavation + $\$5$ for moving per yd$^3$, the cost to move and excavate is estimated at:

$\$25 /\text{yd}^3 \times \text{Total Volume}$
Problem:
A roadway is to be designed on a level terrain. This roadway is 150 meters in length. Four cross sections have been selected, one at 0 meters, one at 50 meters, one at 100 meters, and one at 150 meters. The cross sections, respectively, have areas of 40 square meters, 42 square meters, 19 square meters, and 34 square meters. What is the volume of earthwork needed along this road?

Solution:
Three sections exist between all of these cross sections. Since none of the sections end with an area of zero, the average end area method can be used. The volumes can be computed for respective sections and then summed together.

- Section between 0 and 50 meters:
  \[ V = \frac{A_1 + A_2}{2}L = \frac{40 + 42}{2}50 = 2050 \text{ cubic meters} \]

- Section between 50 and 100 meters:
  \[ V = \frac{A_1 + A_2}{2}L = \frac{42 + 19}{2}50 = 1525 \text{ cubic meters} \]

- Section between 100 and 150 meters:
  \[ V = \frac{A_1 + A_2}{2}L = \frac{19 + 34}{2}50 = 1325 \text{ cubic meters} \]

- Total Volume is found to be:
  \[ 2050 + 1525 + 1325 = 4900 \text{ cubic meters} \]