# Mechanics of Materials-Deflection

## Beam Deflections

The deformation of a beam is usually expressed in terms of its deflection from its original unloaded position. Beams deflect (or sag) under load. Even the strongest, most substantial beam imaginable will deflect under its own weight. Under normal conditions, the actual amount of deflection in floor beams is generally un-noticeable as shown in Figure 1.

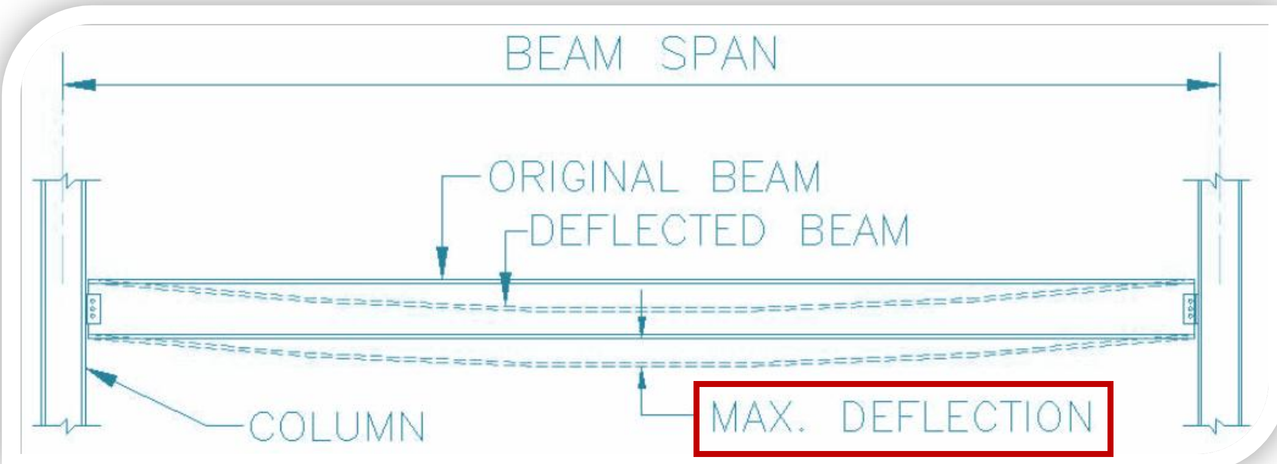



**Figure 1**

**1-Maximum (Actual) deflection** ($\Delta_{max}$)

The engineer calculates the actual deflection (shown in Figure2) of a particular beam or load condition. The factors that need to be considered when calculating deflections are span(L), load(w), beam shape, material properties(E and I) and end fixity(roller, fixed or hinge supports).

**2-Allowable deflections ($\Delta_{allowable}$)**

Studies have shown that excessive deflection in beams causes undesirable effects, such as cracked ceilings and floors as well as vibration.

Building codes (IBC, 2008) typically specify the maximum allowable deflection so as to avoid these problems. The **Maximum-actual** deflections are **compared** against the **allowable deflections** in another check of structural adequacy, sometimes referred to as "**serviceability**" checking.

It is possible that a lightly-loaded beam having a relatively long span

may be adequate in a stress analysis, but fail due to deflection.
Various allowable deflections are shown in *Table 1* below.

**Table 1**. Limits for deflection (Dead + Live)

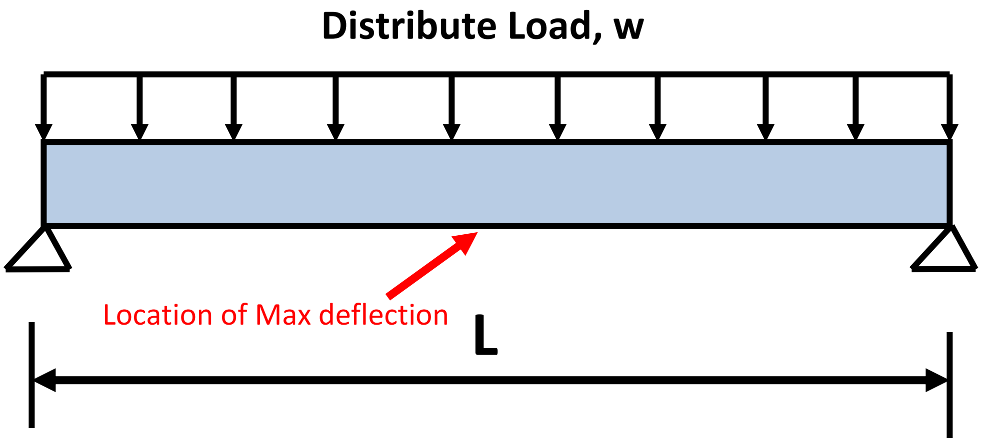
| Type | Allowable deflection, ($\Delta_{allowable}$) |
|---|---|
| (Live load only) | $\frac{L}{360}$ |
| (Dead + Live load) | $\frac{L}{240}$ |
| **Where L is length of the beam** | |

**Beam Loading Conditions**

There are various loading conditions and straight forward equations used to calculate the Max deflection ($\Delta_{max}$) for any beam. The main loading conditions for this morning section that need to be practiced for are, (a)uniformly distributed, (b)concentrated load, (c)combination of uniformly and distributed, (d)two  equally concentrated loads and a(e) cantilever with concentrated load at a free-end as shown below.

## (a)Uniformly distributed Loads

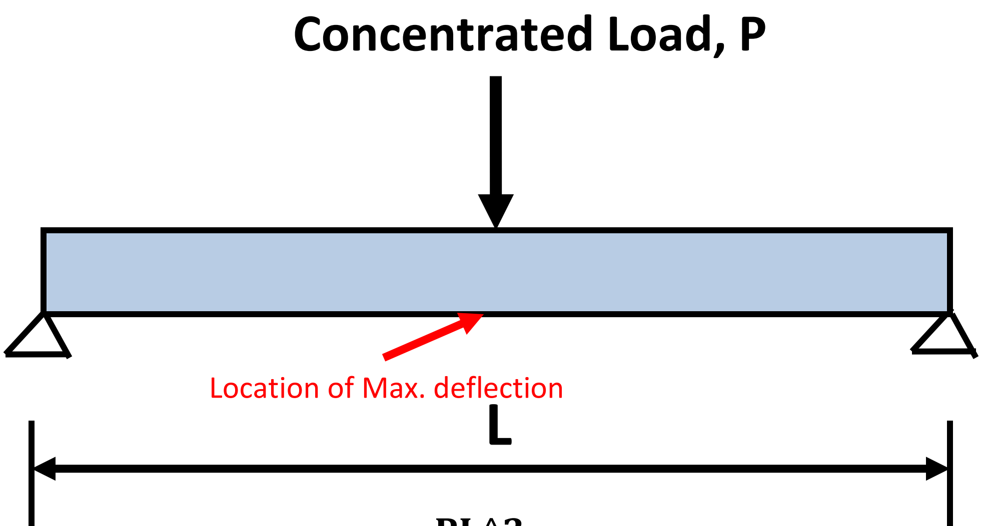
A uniform distributed load is a distributed load that has a constant value, (Example 1lb/ft).




**(Maximum Deflection)** $\Delta_{max} = \frac{5wL^4}{384EI}$ @ mid span

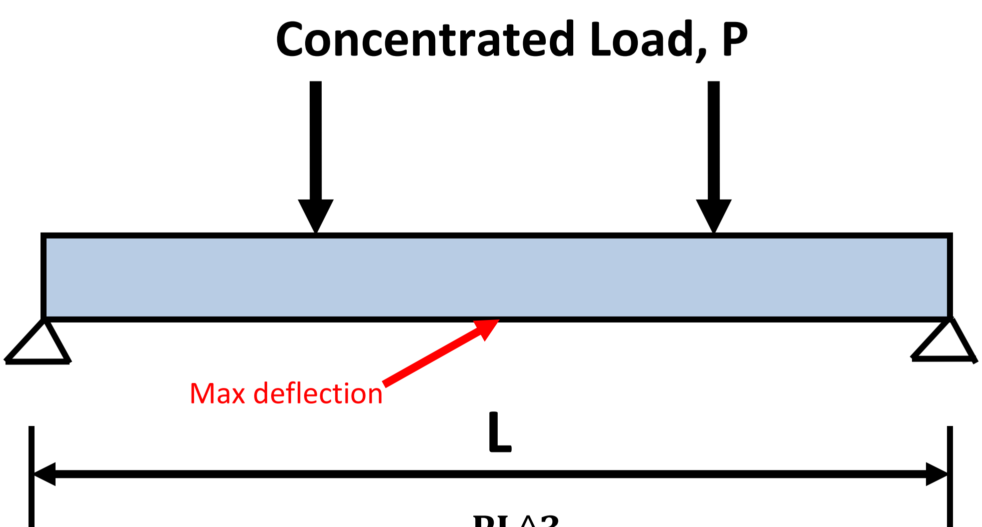
Where E is the modulus of elasticity of the material used,
E for steel = 29500 ksi (**given in exam**)
I is the moment of inertia of the beam shape.
W is distribute load and L, beam length

**(b)Concentrated Load @ center of beam (midspan)**



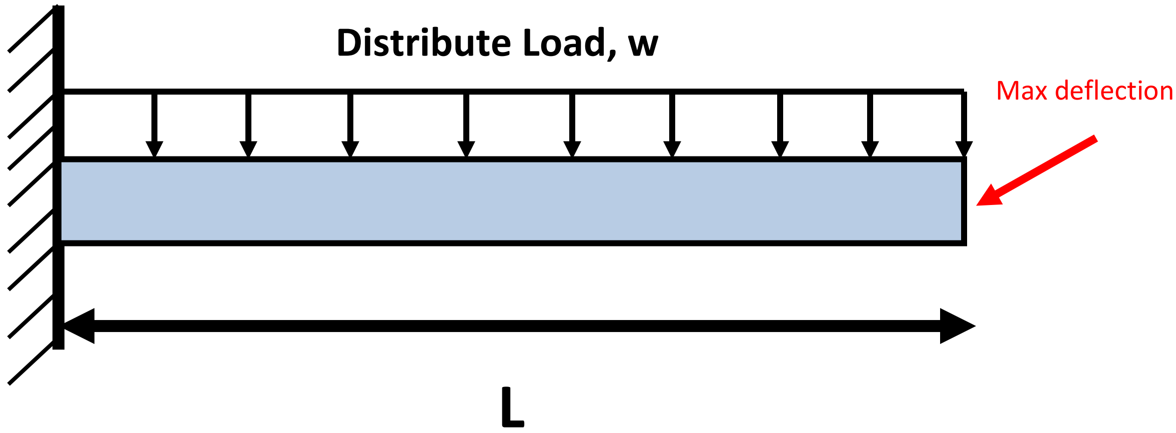

$$\Delta_{max} = \frac{PL^3}{48EI} \text{ @ mid span}$$

**(c)Two equal concentrated loads at 1/3 points**



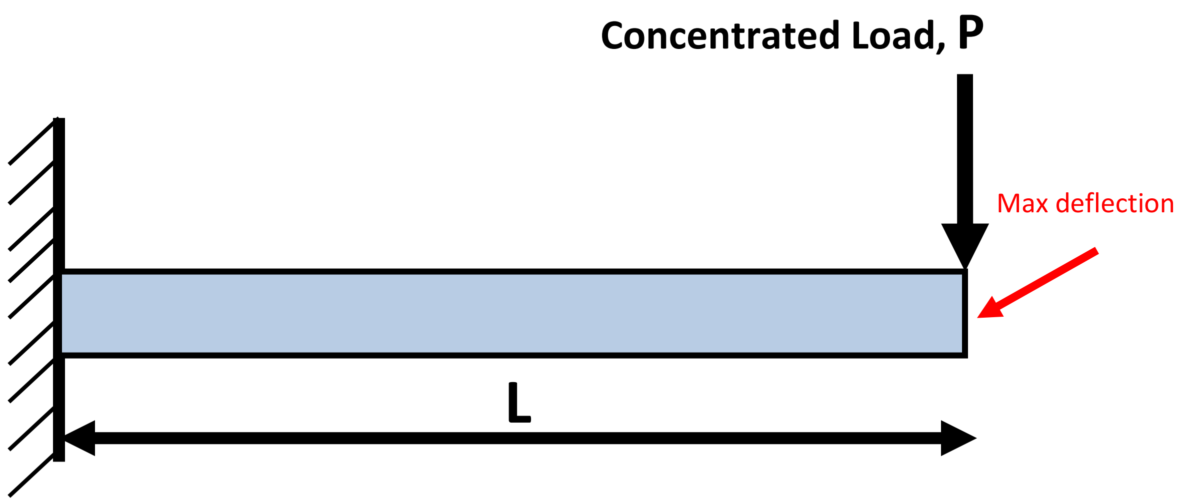

$$\Delta_{max} = \frac{PL^3}{28EI} \text{ @ mid span}$$

## (d)Cantilever with uniform load




$$\Delta_{max} = \frac{wL^4}{8EI} \text{ @ the free end}$$

## (e)Cantilever with concentrated load at free end

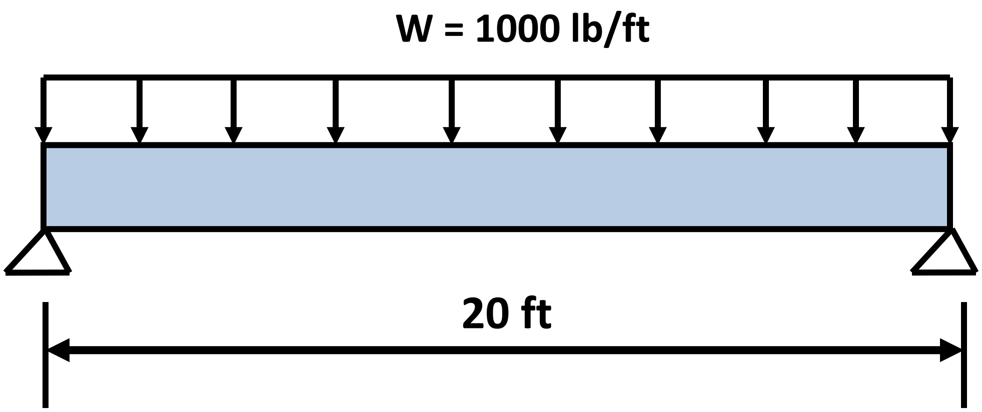



$$\Delta_{max} = \frac{PL^3}{3EI} \text{ @ the free end}$$

## Example problem (1)

**Given:** A simply supported steel beam carries a service uniform load of 1000 lb/ft including the beam weight**,** where E=29500ksi and I=300in$^4$

**Find:** Maximum Deflection @ mid span?




Solution: $\Delta_{max} = \frac{5WL4}{384EI} = \frac{5*\frac{1000lb}{ft}*\frac{ft}{12in}*(20*12)\text{^}4(ft^4)}{384*29500000\left(\frac{lb}{in2}\right)*(300)\quad in4}$= **<u>0.406 inch</u>**

**(A) 0.50inch (B) 0.45inch (C) 0.41inch (D) 0.48inch**

**This is the Maximum deflection**

**<u>NOTES</u>:** Do not forget to convert E from kips to pounds and the length multiplied by 12(1ft=12inch).

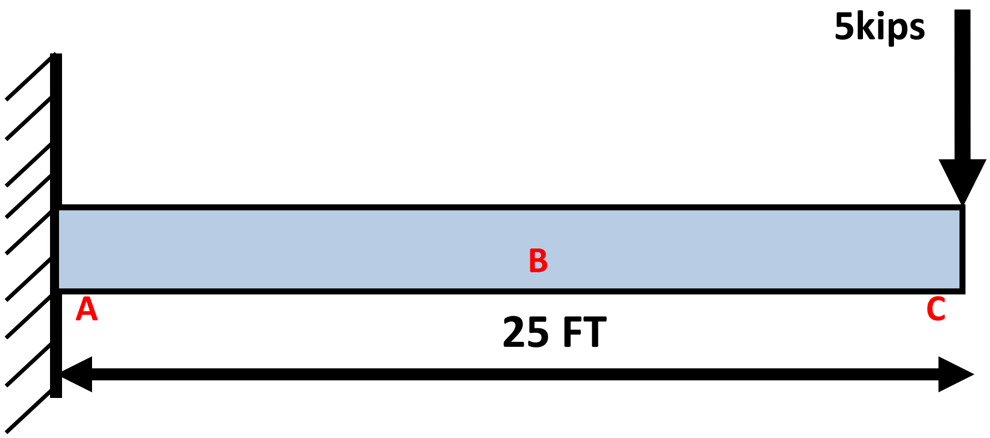
Each problem is **<u>2.5%</u>** of the morning section and it can cost you passing the exam. You might be familiar with the subject or even an expert but the machine counting score, DOES NOT.UNITS UNITS UNITS

## Example problem (2)

**Given:** A cantilever beam with a 5 kips service concentrated load @ the tip of the beam as shown above. The length of beam is 25 ft. while E=29000ksi and I=300in4

**Find:** (a) Maximum Deflection due to the concentrated load in **INCH**?

(b) Where does the maximum deflection occur?




## Solution

(a) $\Delta_{max} = \frac{PL\text{^}3}{3EI} = \frac{5kips*(25^3)ft3}{3*29500ksi*\frac{144in2}{ft2}*300in4\frac{ft4}{(12^4)in4}}$

=0.424FT x12 = **5.01 inch**

(b)Maximum deflection occurs at tip of the beam at point C.

**Example problem (3)**

**Given**: For example problem 1 above.

(a)What is the Allowable deflection in **inches**, if the allowable deflection DL+LL due to is L/240; If the load applied represent the Dead and Live loads, determine if the beam deflection is **acceptable**?

**Solution**: Refer to table 1(pg2) for $\Delta_{allowable}$ = L/240 = $\frac{20*12}{240}$=**1 inch**

since the Actual deflection (0.406in) is **Less** than the Allowable deflection (1inch), It's **ACCEPTABLE.**

**Example problem (4)**

**Given:** Deflection of two beams(1 & 2), similar to case(a) of the uniformly distributed load is to be calculated. If the moment of inertia of beam 1 is three times that of beam 2. Assume w, E and L are the same.

**Find**: (a) What is the Maximum deflection ratio of beam 1 to beam 2?

**Solution** :$\left(\frac{5wL4}{384E(3I)}\right)_{beam1} = \left(\frac{5wL4}{384E(I)}\right)_{beam2}$

$$= \frac{\cancel{5wL^4}}{\cancel{384E}(3I)} \times \frac{\cancel{384E}(I)}{\cancel{5wL^4}} = \frac{I}{3I} = \frac{\mathbf{1}}{\mathbf{3}}$$

Therefore the ratio is 1 to 3; hence, beam2 deflection is 3 times that of beam1.

**Example problem (5)**

**Given:** Deflection of two beams (1 & 2), similar to case (a) of the uniformly distributed load is to be calculated. If beam 1 length is twice that of beam 2. Assume w, E and I are the same.

**Find**: (a) What is the Maximum deflection ratio of beam 1 to beam 2?

Solution: $\Delta_{max}$=( $\frac{5w(2L)^4}{384E(3I)}$ )$_{beam1}$ = ( $\frac{5wL^4}{384E(I)}$)$_{beam2}$ divide beam1 / beam2

= $\frac{\cancel{5w}(2L)^4}{\cancel{384E(I)}}$ x $\frac{384E(I)}{\cancel{5w}L^4}$=$\frac{16L^4}{L^4}$= $\frac{\mathbf{16}}{\mathbf{1}}$, **hence 16:1**