Lecture 24 – Prestressed Concrete

Prestressed concrete refers to concrete that has applied stresses induced into the member. Typically, wires or “tendons” are stretched and then blocked at the ends creating compressive stresses throughout the member’s entire cross-section. Most Prestressed concrete is precast in a plant.

**Advantages** of Prestressed concrete vs. non-Prestressed concrete:

- More efficient members (i.e., smaller members to carry same loads)
- Much less cracking since member is almost entirely in compression
- Precast members have very good quality control
- Precast members offer rapid field erection

**Disadvantages** of Prestressed concrete vs. non-Prestressed concrete:

- More expensive in materials, fabrication, delivery
- Heavy precast members require large cranes
- Somewhat limited design flexibility
- Small margin for error
- More complicated design

**Typical Precast Prestressed concrete members**
Pre-Tensioned Prestressed Concrete:

Pre-tensioned concrete is almost always done in a precast plant. A pre-tensioned Prestressed concrete member is cast in a preformed casting bed. The BONDED wires (tendons) are tensioned prior to the concrete hardening. After the concrete hardens to approximately 75% of the specified compressive strength $f'_c$, the tendons are released and axial compressive load is then transmitted to the cross-section of the member.

**Step 1**

- Tendons tensioned between bulkheads
- Casting bed
- Prestress force $P_s$

**Step 2**

- Fresh concrete placed in bed
- Tendons anchored at “Live” end and “Dead” end

**Step 3**

- Hardened concrete
- Tendons released at “Live” end and “Dead” end creating an axial force along length of precast member
- Prestress force $P_s$
Analysis of Rectangular Prestressed Members:

The analysis of a member is typically done for various stages of loading under **SERVICE LOADS**. Stresses “f” are obtained as follows:

\[
f = \frac{P_s \pm P_s e y}{A_g I_g}
\]

where: 

- \(P_s\) = prestress force 
- \(A_g\) = gross cross-sectional area of member 
- \(e\) = eccentric distance between prestressing tendons and member centroid 
- \(y\) = distance from centroid to extreme edge of member 
- \(I_g\) = gross moment of inertia of member about N.A.

\[
M_u = 0.9 A_{ps} f_{pu} (d_p - \frac{a}{2})
\]

where: 

- \(M_u\) = usable moment capacity of prestressed beam 
- \(A_{ps}\) = area of prestressed tendons 
- \(f_{pu}\) = ultimate tensile strength of prestressing tendon 
- \(\gamma_p\) = factor based on the type of prestressing steel 
  - = 0.40 for ordinary wire strand 
  - = 0.28 for low-relaxation wire strand 
- \(\beta_1\) = 0.85 for concrete \(f'c = 4000\) PSI 
  - = 0.80 for concrete \(f'c = 5000\) PSI 
- \(\rho_p\) = \[\frac{A_{ps}}{b d_p}\]
\[ a = \frac{A_{ps} f_{ps}}{0.85 f'_{c} b} \]
Example

**GIVEN:** The rectangular prestressed concrete beam as shown below. Use the following:

- Concrete $f'_c = 5000$ PSI
- Concrete strength = 75%($f'_c$) at time of prestressing
- $A_{ps} = 3 - \frac{1}{2}$" dia. 7-wire strands @ 0.153 in$^2$ per strand = 0.459 in$^2$
- $f_{pu} = 270$ KSI (using an ordinary 7-wire strand)
- Initial prestress force, $P_s = 70% (f_{pu})(A_{ps})$
- Service dead load, (NOT including beam weight) = 400 PLF
- Service beam weight = 188 PLF
- Service live load = 1500 PLF

**REQUIRED:**
1) Determine the location of the neutral axis and prestress eccentricity “e”.
2) Determine the moment of inertia about the neutral axis, $I_g$.
3) Determine the stresses during prestressing.
4) Determine the stresses during initial applied service beam weight.
5) Determine the stresses due to service applied dead load + live load.
6) Determine the final stresses due to all service loads and prestressing.
7) Determine the maximum actual factored moment on the beam $M_{max}$.
8) Determine the factored usable moment capacity $M_u$ of the beam.
Step 1 – Determine the location of the neutral axis and prestress eccentricity “e”:

Using a datum as measured from the top of the beam:

\[
n = \frac{E_{\text{steel}}}{E_{\text{conc}}} = \frac{29,000,000 \text{ PSI}}{57,000 \sqrt{f'_c} = 5000 \text{ PSI}} = 7.2
\]

\[nA_{ps} = 7.2(0.459 \text{ in}^2) = 3.30 \text{ in}^2\]

\[y_t = \frac{\sum A y}{\sum A} = \frac{(10'' \times 18'')9'' + (3.30 \text{ in}^2)16''}{(10'' \times 18'') + (3.30 \text{ in}^2)} = 9.13''\]

\[y_b = 18'' - 9.13'' = 8.87''\]

\[e = d_p - y_t = 16'' - 9.13'' = 6.87''\]
Step 2 – Determine the moment of inertia about the neutral axis, $I_g$:

$$I_g = \frac{bh^3}{12} + bh \left( y_t - \frac{h}{2} \right)^2 + nA_{ps}(e)^2$$

$$= \frac{(10\text{''})(18\text{''})^3}{12} + (10\text{''})(18\text{''}) \left( 9.13\text{''} - \frac{18\text{''}}{2} \right)^2 + (3.30\text{in}^2)(6.87\text{''})^2$$

$$= 4860 \text{in}^4 + 3.0 \text{in}^4 + 155.7 \text{in}^4$$

$$I_g = 5018.7 \text{in}^4$$

Step 3 – Determine the stresses during prestressing:

$$f = -\frac{P_s}{A_g} \pm \frac{P_s e_y}{I_g}$$

where: $P_s = \text{prestress force}$

$$= 70\%(f_{pu})(A_{ps})$$

$$= 0.70(270 \text{ KSI})(0.459 \text{ in}^2)$$

$$= 86.8 \text{ KIPS}$$

$y = y_t$ for tensile stresses at top of beam

$= y_b$ for compressive stresses at bottom of beam

a) Check stresses at TOP of beam:

$$f_{top} = \text{stress at top of beam}$$

$$= -\frac{P_s}{A_g} + \frac{P_s e_y}{I_g}$$

$$= -\frac{86.8\text{KIPS}}{(10\text{''} \times 18\text{''})} + \frac{(86.8\text{KIPS})(6.87\text{''})(9.13\text{''})}{5018.7\text{in}^4}$$

$$= -0.48 \text{ KSI} + 1.08 \text{ KSI}$$

$$f_{top} = 0.60 \text{ KSI Tension}$$
b) Check stresses at BOTTOM of beam:

\[ f_{\text{bottom}} = \text{stress at bottom of beam} \]

\[ = - \frac{P_2}{A_g} - \frac{P_2 \text{ey}_b}{I_g} \]

\[ = -\frac{86.8\text{KIPS}}{10\text{"} \times 18\text{"}} = \frac{(86.8\text{KIPS})(6.87\text{"})(8.87\text{"})}{5018.7\text{in}^4} \]

\[ = -0.48 \text{ KSI} - 1.05 \text{ KSI} \]

\[ f_{\text{bottom}} = -1.53 \text{ KSI Compression} \]

Step 4 – Determine the stresses during initial applied service beam weight:

\[ f = \pm \frac{M_{\text{beam}}(y)}{I_g} \]

where: \( M_{\text{beam}} = \text{maximum unfactored moment due to beam wt.} \)

\[ = \frac{w_{\text{beam}}(L)^2}{8} \]

\[ = \frac{(188\text{PLF})(22'-0'\text{")}^2}{8} \]

\[ = 11,374 \text{ Lb-Ft} \]

\[ = 11.4 \text{ KIP-FT} \]

\[ y = y_t \text{ for compression in top} \]

\[ = y_b \text{ for tension in bottom} \]

a) Check stresses at TOP:

\[ f_{\text{top}} = - \frac{M_{\text{beam}}(y_t)}{I_g} \]

\[ = -\frac{(11.4\text{KIP}\times FT)(12\text{"}/ft)(9.13\text{")}}{5018.7\text{in}^4} \]

\[ f_{\text{top}} = -0.25 \text{ KSI Compression} \]
b) **Check stresses at BOTTOM:**

\[ F_{\text{bottom}} = \pm \frac{M_{\text{beam}}(y_b)}{I_g} \]

\[ = + \frac{(11.4 \text{KIP} - FT (12''/\text{ft}))(8.87'')}{5018.7 \text{in}^4} \]

\[ f_{\text{bottom}} = 0.24 \text{ KSI Tension} \]

**Step 5 – Determine the stresses due to service applied dead load + live load:**

\[ f = \pm \frac{M_{\text{DL+LL}}(y)}{I_g} \]

where: \( M_{\text{DL+LL}} = \) maximum unfactored moment due to DL+LL

\[ = \frac{w_{\text{DL+LL}}(L)^2}{8} \]

\[ = \frac{(400 \text{PLF} + 1500 \text{PLF})(22'-0'')(22'')}{8} \]

\[ = 114,950 \text{ Lb-Ft} \]

\[ = 115.0 \text{ KIP-FT} \]

\( y = y_t \) for compression in **top**

\( = y_b \) for tension in **bottom**

a) **Check stresses at TOP:**

\[ f_{\text{top}} = -\frac{M_{\text{DL+LL}}(y_t)}{I_g} \]

\[ = -\frac{(115.0 \text{KIP} - FT (12''/\text{ft}))(9.13'')}{5018.7 \text{in}^4} \]

\[ f_{\text{top}} = -2.51 \text{ KSI Compression} \]
b) Check stresses at BOTTOM:

\[ f_{\text{bottom}} = \frac{M_{DL+LL}(y_h)}{I_g} \]

\[ = + \left( \frac{115.0KIP - FT(12'' / ft))(8.87'')}{5018.7in^4} \right) \]

\[ f_{\text{bottom}} = 2.44 \text{ KSI Tension} \]

Step 6 – Determine the final stresses due to all service loads and prestressing:

\[ \begin{align*}
\text{NOTE: All stresses shown have units of KSI}
\end{align*} \]

Step 7 – Determine the maximum actual factored moment on the beam \( M_{\max} \):

\[ M_{\max} = \frac{w_u L^2}{8} \]

\[ w_u = 1.2D + 1.6L \]

\[ = 1.2(400 \text{ PLF} + 188 \text{ PLF}) + 1.6(1500 \text{ PLF}) \]

\[ = 3106 \text{ PLF} \]

\[ = 3.1 \text{ KLF} \]

\[ M_{\max} = \frac{3.1(22'' - 0)^2}{8} \]

\[ M_{\max} = 188 \text{ KIP-FT} \]
Step 8 – Determine the factored usable moment capacity $M_u$ of the beam:

$$
M_u = 0.9A_{ps}f_{ps}(d_p - \frac{a}{2})
$$

where:

$$
f_{ps} = f_{pu} \left(1 - \frac{\gamma_p \beta_1 \rho_p f_{pu}}{f'_{c}}\right)
$$

$f_{pu}$ = ultimate tensile strength of prestressing tendon
= 270 KSI

$\gamma_p$ = factor based on the type of prestressing steel
= 0.40 for ordinary wire strand

$\beta_1$ = 0.80 for concrete $f'_{c}$ = 5000 PSI

$$
\rho_p = \frac{A_{ps}}{bd_p}
$$

$$
= \frac{0.453in^2}{(10'')(16'')}
$$

= 0.00283

$$
f_{ps} = 270KSI \left(1 - \frac{0.40}{0.80} \frac{(0.00283)270KSI}{5KSI}\right)
$$

= 249.4 KSI

$$
a = \frac{A_{ps}f_{ps}}{0.85f'_{c} b}
$$

$$
= \frac{(0.453in^2)(249.4KSI)}{0.85(5KSI)(10'')}
$$

= 2.66"

$$
M_u = 0.9A_{ps}f_{ps}(d_p - \frac{a}{2})
$$

= 0.9(0.453 in^2)(249.4 KSI)(16" - \frac{2.66"}{2})

= 1492 Kip-In

$M_u = 124.3$ KIP-FT $< M_{max} = 188$ KIP-FT $\implies$ NOT ACCEPTABLE